Environmental Taxes and the Choice of Green Technology

Dmitry Krass and Timur Nedorezov
Joseph L. Rotman School of Management, University of Toronto
105 St. George Street, Toronto, ON, M5S 3E6, Canada
krass@rotman.utoronto.ca

Anton Ovchinnikov
Darden School of Business, University of Virginia
100 Darden Boulevard, Charlottesville, VA, 22903, USA
aovchinnikov@darden.virginia.edu

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Abstract

We study several important aspects of using environmental taxes or pollution fines to motivate the choice of innovative and "green" emissions-reducing technologies. In our model, the environmental regulator (Stackelberg leader) sets the tax level, and in response to it a profit-maximizing monopolistic firm (Stackelberg follower), facing price-dependent demand, selects emissions control technology, production quantity and price. The available technologies vary in environmental efficiency, as well as in the fixed and variable costs.

We find that a firm's reaction to an increase in taxes is in general non-monotone: while an initial increase in taxes may motivate a switch to a greener technology, further tax increases may motivate a reverse switch. This reverse effect can be avoided by subsidizing the fixed costs of the green technology; otherwise it could lead to cases under which a given technology cannot be induced with taxes.

We then analyze the socially optimal tax level and the technology choice it motivates. We find that when the regulator is moderately concerned with environmental impacts, the tax level that maximizes social welfare simultaneously motivates the choice of clean technology, resulting in a so-called double dividend. Both low and high levels of environmental concerns lead to the choice of dirty technology. The latter effect can be avoided by subsidizing the capital cost of green technology. Overall, providing a subsidy in conjunction with taxing emissions is generally beneficial: it improves technology choice and increases social welfare; however it may increase the optimal tax level.

1 Introduction

The issues related to the environment and the adoption of "green" technologies have received a large amount of attention over the last several years. Environmentalists won the 2007 Nobel Peace Prize, governments around the world "*placed global warming and greenhouse-gas reduction as one of highest priorities*" (Pelosi, 2008), and business executives put the issues related to the environment at the top of their agendas. According to the November 2007 McKinsey Quarterly global survey*, 41% of senior executives in the U.S. and 53% in Europe believe that the environmental issues will have a large impact on shareholder value over the next 5 years. In this situation managers need to decide whether their firms should adopt new environmentally-friendly technologies that would lower the emissions but may require substantial up-front investments and/or increased production costs. At the same time, environmental regulators are looking for policy instruments to motivate firms make environmentally correct choices. Among other instruments, taxation approach recently gained significant traction among regulators, Hargreaves (2010).

The goal of our paper is to address the interaction between the environmental regulator and the profit-maximizing firm that, as a natural by-product of its production process, emits undesirable pollutant. To do so we consider a Stackelberg game in which a monopolistic firm faces price-dependent demand and in response to the tax level set by the regulator must choose its production quantity and price, as well as the emissions-reducing technology. We assume that a number of different technologies are available to the firm. Technologies vary in their environmental efficiency (the amount of pollutant emitted per unit of the firm's product) and in their fixed and variable costs. In the presence of environmental tax the firm's cost is a sum of its production costs and tax payments, both of which are affected by the technology choice. The regulator, anticipating the firm's production, pricing and technology choices, strategically sets the tax levels to maximize social welfare. In line with earlier works, e.g., Atasu et al (2009), we assume that the social welfare consists of the tax revenue, manufacturer's profit, consumer surplus, net the environmental impact. To translate the environmental impact into monetary units we introduce a parameter measuring the degree of environmental concerns.

*http://www.mckinseyquarterly.com/article_print.aspx?L2=21&L3=0&ar=2077

Our model generates a number of insights with respect to both, the regulator's problem and the firm's optimal response to regulator's actions. In particular, with respect to the firm's response the key insights are:

1. Introducing a tax does *not* necessarily motivate the firm to choose green technology. The "conventional wisdom" is that higher environmental taxes should lead to improvements or upgrades of the existing technology, i.e. the expected reaction from the firm is monotone. This is well summarized by a quote from the Ramseur and Parker's (2009) report to the members of the U.S. Congress: *"if the tax were placed on emissions, entities directly subject to the tax would have an incentive to take actions – e.g., energy efficiency improvements or equipment upgrades - to lower tax payments."*

However, we show that the firm's reaction to taxation may, in fact, be non-monotone: sufficiently high tax rates may induce the choice of dirtier rather than cleaner technologies. We refer to the non-monotonicity as the *negative environmental effect.* The general intuition behind this effect is as follows: higher tax rates increase the variable production costs and therefore price; this in turn reduces the demand and consequently the optimal production quantity. At some tax level the quantity may become small enough so that the total additional profit associated with switching to the cleaner technology may not be sufficient to offset the fixed costs associated with the acquisition and operation of this technology.

2. The negative environmental effect disappears if all technologies have the same fixed cost. In that case the technology choice does become monotone: higher tax rates motivate the choice of greener technology, as the conventional wisdom suggests. We note that the monotone reaction has been also established by other authors, see Proposition 2 in Requate (1998) and Lemma 1 in Amacher and Malik (2002); our result is established under somewhat more general assumptions on the demand function.

The practical importance of this result is that by subsidizing the fixed costs of green technologies, the regulator makes taxation mechanism more effective in motivating the technology choice, suggesting a combined taxation-subsidy strategy. As we discuss later, this also has important implications for the regulator's problem of maximizing social welfare.

3. The problem with $k \geq 3$ technologies has some fundamental differences from the 2-technology setting. In particular, if the firm has three or more technologies to choose from, then the regulator may not be able to motivate the choice of some desirable technology, even if that technology can be induced in a pair-wise comparison with any other technology. This suggests that a new technology must satisfy certain efficiency requirements (particularly with respect to the associated fixed costs) before it becomes possible to induce it through environmental taxation. This finding enriches the current literature which is typically restricted to the two-technology case (e.g., "conventional" vs. "innovative" technologies in Requate (1998), "clean" vs. "dirty" technologies in Amacher and Malik (2002), etc.).

With respect to the regulator's problem, our paper generates the following additional insights:

4. There exists a finite set of tax rates, T, of size $O(k)$, such that the social welfare-maximizing (optimal) tax rate belongs to that set. Thus, the optimal tax rate, which is a solution to a highly non-linear and discontinuous welfare optimization problem, can be found relatively easily.

5. It may be socially optimal to select the tax rate that does not motivate the choice of cleaner technology. In fact, our model shows that it is sometimes optimal to set the tax rate to the highest possible value that does *not* motivate the technology switch. This effect arises because of the conflicting objectives of the regulator: the desire to reduce emissions while receiving tax revenues resulting from pollution; see Chung (2007) and Murphy (2009) for two practical examples of how regulators are explicitly considering this tradeoff. Although this effect may not be technically "negative", as the social welfare is actually optimized at that point, we nevertheless regard such cases as undesirable from the environmental and ethical perspectives. After all, if the stated goal of environmental tax is to induce the choice of greener technologies, the hard-to-quantify political cost of maintaining taxes that actually assure continuing use of dirtier technologies may be quite high.

6. High level of societal environmental concerns do *not* necessarily lead to motivating green technology choice. Again, the conventional wisdom would suggest that increasing awareness of environmental issues within a society should lead to public policy that induces a cleaner technology. However, our model shows that an increase is the societal environmental concerns, may lead to overly high tax rates that, in fact, demotivate the firm from choosing the cleaner technology. This effect is related to the negative environmental effect of the firm's reaction discussed earlier. We also show that this effect can also be avoided if the regulator subsidizes the fixed cost of the cleaner technology. This leads to another important finding.

7. Providing fixed cost subsidy leads to an increase in social welfare. More precisely, we show that by subsidizing the fixed costs of green technology, the regulator improves the technology choice and increases the social welfare. The former effect occurs because due to subsidy the region over which the clean technology is selected expands. This improvement in technology choice contributes to the increase in the social welfare both directly, by reducing the environmental impact, as well as indirectly, by reducing the equilibrium price and hence increasing consumer surplus. We note that the increase in welfare does not happen for all levels of environmental concerns: there is no increase when environmental concerns are either very low (in this case the firm chooses the dirty technology regardless of subsidies) as well as for some medium level of environmental concerns (in this case the socially optimal tax rate induces the choice of the cleaner technology even without subsidies). However, for other levels of environmental concerns there is a non-zero increase in welfare due to subsidies.

We also examine three types of solutions: the "fully coordinated" solution where a central authority controls the tax rates, prices and the production quantity, the "subsidy-coordinated" solution where the regulator sets the tax rates and subsidizes the fixed costs of greener technologies, and the "decentralized" solution which reflects our original model where the regulator sets the tax rates while the firm controls the production quantity. We note that the "fully coordinated" solution is sometimes known as "first best" in the economics literature. In this regard our main finding are as follows:

8. For some optimal tax rates the centralized, subsidy-coordinated and decentralized solutions coincide. Our results show that the set of tax rates where a given technology is inducible consists of the union of intervals (provided the technology is inducible). When the optimal tax rate corresponding to this technology choice falls within the inducibility set of a given technology, the centralized, subsidy-coordinated and decentralized solutions coincide – i.e., the level of social welfare achieved with the decentralized solution is identical to the level achieved by the centralized solution.

9. The centralized and subsidy-coordinated solutions coincide provided the level of societal environmental concerns is high enough. The implications of this result are that if the environmental awareness can be increased above a certain threshold, a combination of fixed cost subsidies and environmental taxes will always achieve the highest possible level of social welfare, without the need for the regulator to directly control firm's production quantity and prices. This result shows the efficiency and robustness of the combined subsidy-environmental taxation approach.

We close introduction with two further remarks about our model. First, in this paper we consider the case of a monopoly. A case with an oligopoly[†] is much more complex and is discussed in a follow-up paper. Second, we note that unlike most of the economics literature that considers emission reduction as the sole goal of the environmental policy, we consider the inducement of cleaner technology as an equally, if not more, important goal. This view is largely driven by the operational and practical concerns: while instituting very high environmental taxes that lead to reduction in economic output with accompanying reduction in emissions is often seen as politically unpalatable, programs aimed at motivating the choice of a cleaner technology are much more popular. Rebates or tax inducements offered in many jurisdictions for the purchase of hybrid cars, energy efficient appliances, solar panels, or other similar clean technologies are all examples of such programs - adopters are rewarded for acquiring the clean technology, irrespective of whether the resulting emissions go up or down. This focus on the inducement of cleaner technology suggests that the most desirable outcome from the regulator's perspective is a "win-win" situation when the tax rate that maximizes social welfare also induces the cleanest available technology. We refer to such cases as *double dividends*; we note that our interpretation of double dividend is somewhat different from that used in economics - the latter emphasizes total emission level rather than technology choice, e.g., see Goulder (1995) for the "classical" definition. Our results indicate that double dividends do occur "naturally" under for certain parameter values, but are much more frequent under the subsidy-environmental taxes regime, providing yet another reason for using subsidies to offset fixed costs of new technologies.

The remainder of the paper is organized as follows. Section 2 positions our work in the body of existing literature. The model is formulated in Section 3. Sections 4 and 5 study the firm's and the regulator's problems, respectively. Section 6 discusses the role subsidies.

[†]The monopoly case has some similarity with the Bertrand oligopoly; intuitively, the competitor with the technological advantage can undercut all other firms and become a monopolist for the range of tax rates over which its technology choice is optimal. In the Cournot oligopoly all firms charge the same price.

Section 7 summarizes the paper and discusses future research. The paper is accompanied by an (online) appendix.

2 Literature Review

Our paper is related to two streams of literature: one in operations management and another in economics. In the operations literature, a number of researchers addressed environmental problems with respect to remanufacturing, e.g., Mujumder and Groenvelt (2001), Debo et al. (2005), Ferguson and Toktay (2006), Souza et al. (2007). Within this domain, several authors address the product take-back legislation, e.g., Atasu et al. (2009), Atasu and Subramanian (2009). These works consider a tax that is charged per unit sold and analyze the incentives this legislation provides for manufacturers to design products to be more recyclable. Our work differs from the above in many dimensions; most importantly, the question is broader – we study production and pollution in general, not just recycling/remanufacturing. At the same time, our model is similar to the above in the sense that we too consider a managerially relevant framework that takes into account operational details of firms' technology choice decisions.

In the economics literature, e.g., see Jaffe et. al. (2002) and Requate (2006) for recent reviews, the questions addressed are more similar to ours, but the framework is very different. Most importantly, the extensive body of economics literature tends to assume away some important operational details of firms' technology choice decisions, while we consider them explicitly.

Specifically, our approach is differentiated from the previous research along the following four dimensions: (i) discrete technology choices with $k \geq 2$ technologies, (ii) non-zero fixed (setup, acquisition, installation) costs associated with each technology, (iii) profit maximization under price-sensitive demand rather than cost minimization choices, and (iv) tax revenue is a part of regulator's social welfare objective. We discuss those in sequel below.

First, a rather common assumption in the economics literature is that a firm's cost of reducing (abating) emissions is given by a continuous twice-differentiable function, e.g., see the fundamental work of Barnett (1980), or a recent treatment by Requate (2006). This implies that a continuum of technologies is available and the firm is deciding on the cost of abating emissions aka "technology." In many cases, however, the managerial decision is very different: the firms must decide whether to invest or not in a given technology from a certain set of available technologies – a discrete choice that underlies our modeling framework. The implicit assumption here is that such technologies are available in the market for the firm to purchase, which we believe is reasonable given the rapidly maturing market for clean technologies, e.g., see Americanventuremagazine.com (2007).

We note that several economics papers do consider discrete technology choices, e.g., Requate (1998), Amacher and Malik (2002) and Fisher et. al. (2003). Large differences between these papers and ours come in the way we consider revenues and costs.

With respect to costs, a rather common assumption in the economics literature is that the set-up costs for the new technology is equal to zero, e.g., Requate (1998), Montero (2002). We assume the opposite: in order to use a technology, the firm must incur a fixed cost of purchasing that technology, installing equipment, etc. Such an assumption is certainly much more realistic, but it also complicates the analysis because the firm's cost and profit becomes non-differentiable, and possibly discontinuous. The inclusion of the fixed costs is a very important feature of our model; it plays an important role in establishing the non-monotonicity of technology choices and the negative environmental effects.

As noted in the introduction, if all technologies have the same fixed cost then firm's response becomes monotone. Requate (1998) establishes monotone response, which occurs because fixed costs are not considered. His model is similar to ours in this case; a slight advantage of our result is that it holds for any demand function. Amacher and Malik (2002) consider fixed cost, but do not consider revenues, and thus also establish monotone response. Note however, that none of those papers discuss the non-monotone response and its implications that we show to exist.

With respect to revenues, a third rather typical assumption is to frame firm's reaction to environmental regulation as a cost minimization problem, see e.g., Amacher and Malik (2002), Fisher et. al. (2003). In our model the firm maximizes profit by selling its product in a market with price-sensitive demand: in practice managers worry about the effects of increased costs on price and consequently on the quantity demanded. The latter is particularly salient because in order to cover the fixed costs of a given technology the firm may need to have a large enough production quantity.

The fourth common assumption is that the regulator is not considering tax revenue as part of social welfare, e.g., Barnett (1980), Requate (1998, 2006). We assume the opposite, and in that sense our model is closer to the operations management papers, such as Atasu et. al. (2009). Effectively by definition, *"governments impose taxes is to raise revenue to fund various objectives or services..."*, Ramseur and Parker (2009), thus we agree with Atasu et. al. (2009) that tax revenue should be included in the regulator's objective explicitly and consider that in our model. Doing so contributes to a number of findings that we discussed in the introduction.

Finally, in addition to the literature discussed above, there are several more streams of literature that are related. The first stream discusses whether it "pays to be green" based on the empirical data, e.g., King and Lenox (2001, 2002). Our paper is obviously different because it presents a model, but at heart we study a similar question: if it "pays" the firm in our model will choose the green technology, and otherwise it will not. Another stream of literature considers endogenous environmental innovation through R&D investments, e.g., Milliman and Prince (1989), Carraro and Topa (1995), Montero (2002). The fundamental difference between our paper and these works is in the discrete nature of technology choice: we assume that the firm is purchasing technology in the market, they assume that the firm is developing the technology endogenously (this effectively implies a continuum of available technologies, with some function that translates R&D dollars into emissions reduction and cost). Discreetness also differentiates our work from Carraro and Soubeyran (1996). Their interpretation of "technology choice" is very different from ours: Whereas we consider discrete technology alternatives (e.g., adopt technology 1 or 2?), they assume that the firm possesses both technologies and consider the problem of allocating production (capacity utilization) between the plants with technology 1 and plants with technology 2.

To summarize, our paper is differentiated from those in the literature in many dimensions. The question we consider is closer to the works in economics, but the framework we use is closer to the works in operations. In terms of key differentiating assumptions, on the firm's side our paper captures the discrete nature of firm's technology choice decisions, incorporates the fixed costs, considers general number of technologies and price sensitive demand. On the regulator's side it includes tax revenue considerations explicitly. We discuss the firm's model next, and the regulator's model in Section 5.

3 The Model

We consider a Stackelberg game between one profit-maximizing price-setting firm producing a particular product, the follower, and a regulatory agency, the leader, that has the power to set the environmental tax or pollution fine level. Pollution takes the form of an undesirable by-product of the production process, and the amount of pollutants released is proportional to the production quantity. We use $x \geq 0$ to denote production quantity. The environmental tax (or fine) level $t \geq 0$ is charged per unit of pollutant emitted into the environment.

The firm has a choice of a finite number of emissions-reducing technologies numbered $1, \ldots, k$. Technology i is described by three parameters: (Θ_i, ξ_i, S_i), where

$\Theta_i \geq 0$ is the one-time fixed (installation, purchase, acquisition, etc.) cost of technology i

$\xi_i \geq 0$ is the variable operating cost (assessed per unit of product) of technology i

$S_i \geq 1$ is the environmental effectiveness parameter of technology i, where x/S_i represents the amount of emissions if the production is set to x.

As an example of using such a triplet to describe the firm's technology choices, consider a firm that is deciding whether to continue operating without emissions-control technology (note this is equivalent to choosing a technology with $\Theta_1 = 0$, $\xi_1 = 0$ and $S_1 = 1$, we will refer to such technologies as *null*), or invest $\Theta_2 = \$120$ million in installing the equipment that will reduce emissions by 95% (i.e., with $S_2 = 20$) at an incremental cost of \$10 per ton of processed waste (i.e., with $\xi_2 = \$10$ times a coefficient translating output into waste[‡]).

We will assume w.l.o.g. that $S_i < S_{i+1}$ for $i = 1, \ldots, k-1$, i.e., higher-indexed technologies are more effective in terms of emissions control. It is natural that this additional effectiveness comes at a cost – thus either $\Theta_{i+1} > \Theta_i$ or $\xi_{i+1} > \xi_i$ (or both) must hold: otherwise technology i is less effective and more expensive than technology $i+1$, and therefore can be dropped from consideration. Beyond that, our description of technologies is very general. Specifically, we do not assume any functional relationship between effectiveness and costs; doing so would result in a major loss of generality without much benefit to the analysis.

Let $C_i(x, t)$ be the firm's cost function. We assume a "fixed plus variable cost" structure, where the fixed cost depends on the selected technology, i, and the variable cost depends

[‡]This example is adopted from Ovchinnikov (2009).

on the selected technology and tax level t. Specifically, let $K, c \geq 0$ represent the baseline fixed and variable production costs of the null technology, respectively, which are incurred irrespective of the emissions-reduction technology choice. Then the total fixed cost is $K + \Theta_i$ and variable cost is $c + \xi_i + t/S_i$, leading to the following production cost function:

$$C_i(x,t) = (K + \Theta_i)I_{x>0} + (c + \xi_i + t/S_i)x, \tag{1}$$

where I_Φ is the indicator function for set Φ. Note that the firm always has a choice to shut down incurring no production costs.

To compute the firm's profit, let p be the price that the firm charges per unit and let $D(p)$ be the demand function faced by the firm. Note that at equilibrium the firm will select the production quantity and price such that $x = D(p)$. Thus, for a given tax level t, technology choice i, and price p, the firm's profit is given by:

$$Pr_i(p,t) = pD(p) - C_i(D(p),t). \tag{2}$$

The firm's best response strategy to a given tax level t can be viewed as a two-stage problem. First, for each technology choice i the firm optimizes its market response $p_i^*(t)$ by solving:

$$Pr_i(t) = \max_{p \geq 0} Pr_i(p,t).$$

Second, the firm optimizes its technology choice $i^*(t) = argmax_{i \in \{1,\ldots,k\}} Pr_i(t)$. The firm's strategy is then described by the pair $\{i^*(t), p^*_{i^*(t)}(t)\}$ because $x^*_{i^*(t)}(t) = D(p^*_{i^*(t)}(t))$. This can be regarded as the firm's reaction function to the tax level t in the leader-follower Stackelberg game.

Since our interest lies primarily in the effect of taxation on technology choice, we are particularly interested in taxation levels (when they exist) which make a given technology choice optimal. We make the following definition.

Definition 1 Technology $i \in \{1,\ldots,k\}$ is said to be *inducible* if there exists $t \geq 0$ such that $i^*(t) = i$ and $p^*_{i^*(t)}(t) > 0$, i.e., the firm's optimal reaction to taxation level t is technology choice i and some positive price and consequently, production quantity.

In what follows we discuss the the conditions for inducibility of a given technology. While some of our results apply to general demand functions, to simplify exposition for the most part we will assume a constant elasticity demand function given by

$$D(p) = Ap^{-\delta}, \tag{3}$$

where $\delta > 1$ is the elasticity of demand and $A > 0$ is the scaling constant, denoting, for example, market size. All our results hold for the linear demand function $D(p) = a - bp$; see Appendix.

In view of (1, 2 and 3), for a given technology i and tax level t, the optimal price is given by

$$p_i^*(t) = \delta(c + \xi_i + t/S_i)/(\delta - 1), \tag{4}$$

the optimal quantity is given by

$$x_i^*(t) = \bar{A}(\delta - 1)(c + \xi_i + t/S_i)^{-\delta}, \tag{5}$$

where $\bar{A} = A(\delta - 1)^{(\delta-1)}/\delta^\delta$, and the associated optimal profit is

$$Pr_i(t) = \bar{A}(c + \xi_i + \frac{t}{S_i})^{1-\delta} - (K + \Theta_i). \tag{6}$$

Note that $Pr_i(t)$ is a decreasing convex function of t. It is easy to show that $Pr_i(t) > 0$ iff $0 \leq t < t_i^{lim}$, where $t_i^{lim} = S_i \left[\left(\frac{\bar{A}}{K+\Theta_i} \right)^{1/(\delta-1)} - c - \xi_i \right]$. Let $t^{lim} = \min_{i \in \{1,\ldots,k\}} t_i^{lim}$.

We make the following assumptions:

Assumption 1 *We assume that* $t^{lim} > 0$.

Assumption 2 *We assume that* $Pr_1(0) \geq Pr_2(0) \ldots \geq Pr_k(0)$.

The first assumption holds w.l.o.g. and helps us to avoid trivial cases. If $t^{lim} \leq 0$ then there is some technology $j \in 1, ..., k$ that is never selected, and can therefore be removed from consideration. Similarly, we can restrict attention only to $t \in [0, t^{lim}]$. The second assumption states that for any pair of technologies i, j with $i < j$ (i.e., j is cleaner than i), the cleaner technology j is not strictly preferred at the $t = 0$ tax level, since otherwise the taxes are not required to *motivate* a switch to the cleaner technology; in fact taxes can only motivate a switch away from it. Thus, from the point of view of our paper, this is not an interesting case (in fact, Assumption 2 can be relaxed to only require that $i^*(0) \neq k$ with no significant affect on the results, but at the expense of more cumbersome notation later in the paper).

4 Firm's Problem: Critical Taxation Levels

In this section we analyze the optimal technological choice made by the firm in response to taxation level t. In particular, we seek to answer whether taxes can be used to induce the switch to a cleaner technology. We start our analysis by focusing on the 2-technology case, and then extend it to the more general $k > 2$ case.

4.1 The $k = 2$ case: Pairwise Comparison of Technology Choices

Let $\Delta(t)$ be the difference in optimal profits:

$$\Delta(t) = Pr_2(t) - Pr_1(t). \tag{7}$$

Thus, if $\Delta(t) > 0$, then the cleaner technology 2 would be selected at tax level t, and vice versa. Note that by Assumption 2, $\Delta(0) < 0$ and thus technology 2 is inducible if and only if $\Delta(t) \geq 0$ for some $t \in [0, t^{\lim}]$.

With respect to fixed and variable costs, any two technologies fall into one of the following three categories:

$\Theta_1 > \Theta_2$, $\xi_1 > \xi_2$: This case is trivial: technology 2 is cleaner and has lower fixed and variable costs. Technology 1 can be removed from consideration.

$\Theta_1 \leq \Theta_2$, $\xi_1 > \xi_2$: This is not an interesting case: from (1) technology 2 has largest variable cost advantage at $t = 0$ and from (5) the optimal production quantity is largest when $t = 0$. Thus the cleaner technology 2 is either already selected at $t = 0$ and higher taxes can only motivate the choice of dirty technology, or as we assumed to avoid that "silly" situation (Assumption 2), it will never be selected.

$\xi_1 \leq \xi_2$: This[§] is the only interesting case and we consider it in detail below.

When $\xi_1 \leq \xi_2$, $\Delta(t)$ is quasi-concave with a unique maximum at:

$$t^{max} = \min\left[\frac{(c+\xi_2)(S_2/S_1)^{1/\delta} - (c+\xi_1)}{1/S_1 - 1/S_2(S_2/S_1)^{1/\delta}}, t^{\lim}\right], \tag{8}$$

which leads to the following result:

[§]Condition $\xi_1 \leq \xi_2$ is in fact rather intuitive: For instance, the (non-fuel related) variable cost of operating a hybrid car is certainly higher than that of a regular car: a hybrid car has many parts and devices that a regular car does not have, as a result $\xi_{hybrid} > \xi_{regular}$.

Observation 1 *In the 2-technology case, the cleaner technology 2 is inducible if, and only if,* $\xi_1 \leq \xi_2$ *and* $\Delta(t^{max}) \geq 0$.

Further, as noted above, $\Delta(t)$ is increasing for $t \in [0, t^{max}]$ and decreasing for $t \in [t^{max}, t^{\lim}]$. Since $\Delta(0) < 0$, this implies that equation $\Delta(t) = 0$ has at least one, and possibly two roots on $[0, t^{\lim}]$. Let us denote these roots by t_{12}^{crit}, t_{21}^{crit} (the sequence of indices in t_{ij}^{crit} signifies that a switch occurs from technology i to j), with $t_{21}^{crit} = t^{\lim}$ if the second root does not exist. We refer to these taxation levels as *critical*, because they correspond to changes in the technology choice. In general three regions can be identified:

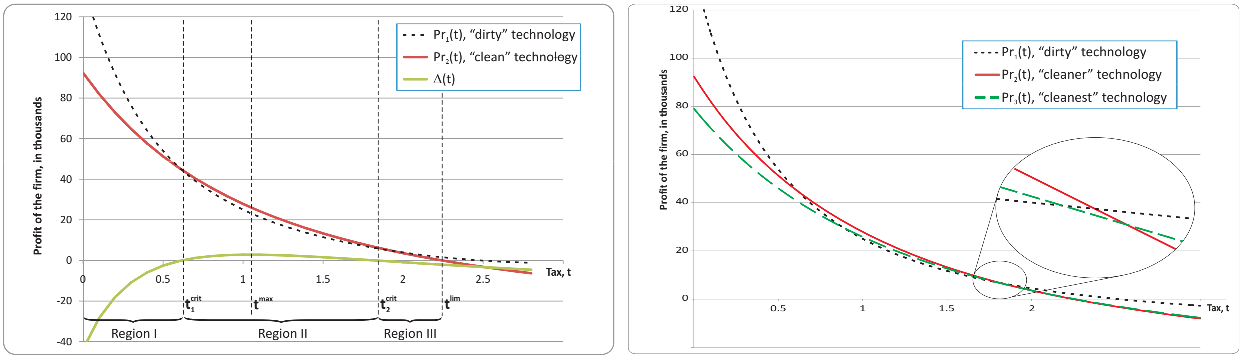
$$\begin{array}{lll} \text{Region I:} & 0 < t < t_{12}^{crit} & \text{Technology 1 (``dirty'') is preferred} \\ \text{Region II:} & t_{12}^{crit} < t < t_{21}^{crit} & \text{Technology 2 (``clean'') is preferred} \\ \text{Region III:} & t_{21}^{crit} < t & \text{Technology 1 (``dirty'') is preferred,} \end{array} \tag{9}$$

with the firm indifferent between two technologies for $t \in \{t_{12}^{crit}, t_{21}^{crit}\}$. The following example illustrates the discussion.

Example 1 ***Existence of the three regions.*** Suppose the market size $A = 1,000,000$, demand elasticity $\delta = 3$, and baseline fixed and variable production costs are given by $K = 12,000$ and $c = 1$, respectively. Suppose technology 1 is null (i.e., $S_1 = 1$, $\xi_1 = \Theta_1 = 0$) and technology 2 has the following parameters: $S_2 = 2$, $\xi_2 = 0.1$, $\Theta_2 = 18,000$. That is, the cleaner technology 2 reduces emissions (and tax obligations) twofold, has installation costs of $18,000$ and increases variable production costs by 0.1 per unit (i.e., by 10 percent).

Using expressions derived above, it can be computed that $t^{lim} = 2.244$, $t_{12}^{crit} = 0.616$ and $t_{21}^{crit} = 1.828$. The resulting functions $Pr_1(t), Pr_2(t)$ and $\Delta(t)$ are illustrated on Figure 1 (a). Here the dirty null technology 1 will be selected for $0 < t < 0.616$ (region I) and $t > 1.828$ (region III), while the clean technology 2 is only preferred for $0.616 < t < 1.828$ (region II).

The existence of the second root t_{21}^{crit} and Region III may, at first, appear counter-intuitive: since increasing tax t has the effect of reducing the variable cost of technology 2 relative to technology 1, why should increasing taxes beyond t_{21}^{crit} make technology 1 more advantageous? The reason this *negative environmental effect* may arise at higher taxation levels is in the fixed costs. Increasing taxes has two effects: decreasing the variable production cost of technology 2 relative to technology 1 (since $t/S_1 > t/S_2$), but also increasing the firm's total



(a) (b)

Figure 1: Cases with 2 and 3 technologies. In (a), the existence of three critical regions, illustration for Example 1. In (b), the cleanest technology is not inducible, illustration for Example 2.

production costs. While the first effect makes technology 2 more attractive as t increases from 0 up to t^{max}, after the level t^{max} is crossed, the second effect takes over. The increase in total production costs increases the optimal market price, which leads to lower optimal demand and hence lower production quantity. This, in turn, reduces the total profit of the firm. Since the cleaner technology 2 has higher fixed cost Θ_2, when the production level is sufficiently low, the relative variable cost advantage gained through the reduction in environmental tax obligations is no longer sufficient to offset higher fixed costs.

We note that the existence of Region III is fairly common. Extending the above example to allow $\delta \in (1, 5]$, $S_2 \in [1, 5]$, $\xi_2 \in [0, 1]$, and $\Theta_2 \in [0, 20000]$ we observed that Region III is non-empty in approximately 27% of problem instances. This may have important practical consequences: Since the exact values of critical taxation levels depend on problem parameters which are unlikely to be known with certainty by the regulator, an increase in the environmental tax rate may inadvertently push the taxation level into Region III, thus triggering a move away from the cleaner technology. We discuss this issue in more detail in Section 5.

Overall, Region III is not empty and the negative environmental effect exists when the clean technology has high installation costs, as is shown in the following Proposition:

Proposition 1 *Assume values of $\delta > 1$, $K, c > 0$, $0 \le S_1 < S_2$ and $0 \le \xi_1 < \xi_2$ are given and A is sufficiently large. Then, there exist $0 \le \Theta_1 < \Theta_2$ such that $t_{21}^{crit} < t^{\lim}$, i.e., Region III is not empty.*

From the proof of the above Proposition, we can establish the following Corollary:

Corollary 1 *The value of* t_{12}^{crit} *is non-decreasing in the fixed cost* Θ_2 *and the value of* t_{21}^{crit} *is non-increasing in* Θ_2*. Therefore the size of Region II, given by* $t_{21}^{crit} - t_{12}^{crit}$*, is non-increasing in* Θ_2*.*

The preceding result says that as the fixed cost Θ_2 decreases, Region II expands. This suggests that subsidies given to the firm in (even partial) compensation of the fixed cost of the cleaner technology may be effective in expanding the region where the cleaner technology is chosen.

We note that all of the structural results discussed above hold for the case of linear demand functions – see Appendix. We also conjecture that the results, in fact, hold for substantially more general demand and productions functions – however, the resulting behavior of the $\Delta(t)$ function may be more complicated; in particular there may be more than two critical taxation values present (i.e., there may be more than two alternating regions where each technology is preferred).

4.2 Technology Choice with $k \geq 3$ Technologies

The case with more than 2 technologies is similar, but somewhat more complex, than the 2-technology case. We use the same notation as in the preceding section, adding indices designating technologies being compared. For example, $\Delta_{ji}(t) = Pr_j(t) - Pr_i(t)$ is the difference in profitability of technologies j and i at the taxation level t, and t_{ji}^{max} is the tax level which maximizes $\Delta_{ji}(t)$.

Our primary interest is in analyzing when the cleanest technology k is the most profitable one. We first note that that Proposition 1 immediately implies the following result.

Corollary 2 *The following conditions are necessary for technology* k *to be inducible:*

$$\Delta_{ki}(t_{ki}^{max}) \geq 0 \quad \textit{for all} \quad i = 1, \dots, k-1. \tag{10}$$

However, these pairwise comparisons are not sufficient, as the example below demonstrates.

Example 2 ***Non-inducibility in the 3-technology case.*** With the same parameters as in Example 1, assume there are now three technologies available: technology 1 is the null technology ($S_1 = 1, \xi_1 = 0, \Theta_1 = 0$), technology 2 is the "cleaner" technology with parameters $S_2 = 2, \xi_2 = 0.1, \Theta_2 = 18000$ (i.e., equivalent to technology 2 in Example 1), and the cleanest technology 3 that has the following parameters: $S_3 = 2.2, \xi_3 = 0.16, \Theta_3 = 19000$.

As in Example 1, it is easy to compute that $t_{13}^{crit} = 0.884$ and $t_{31}^{crit} = 1.790$. By analyzing $\Delta_{23}(t)$ we observe that it only has one root with $t_{23}^{crit} = 1.986$, by our convention therefore the second root $t_{32}^{crit} = t^{lim} = 2.244$. That is, technology 3 is better than technology 2 for $t > 1.986$, but in that range it is worse than technology 1. On the other hand, when $t \in (0.884, 1.790)$, technology 3 is better than 1, but is worse than 2. Thus, technology 3 is pairwise inducible against any other technology, but is not inducible at any tax level against both technologies 1 and 2 jointly; see Figure 1 (b) for illustration.

To rigorously define the inducibility region for technology $j \in \{1, \ldots, k\}$ we proceed as follows. From (9), for $i < j$ we have $t_{ij}^{crit} \le t_{ji}^{crit}$ and technology j is at least as profitable as i iff $t \in [t_{ij}^{crit}, t_{ji}^{crit}]$. On the other hand, if $j < i$ then $t_{ji}^{crit} \le t_{ij}^{crit}$ and j is at least as profitable as i iff $t \in [0, t_{ji}^{crit}] \cup [t_{ij}^{crit}, t^{lim}]$. Since technology j is cleaner than $i < j$ and dirtier than $i > j$, if we intersect the corresponding intervals and designate them with indices (C) and (D) (depending on whether j is cleaner or dirtier, respectively), we obtain the following result:

Lemma 1 *The inducibility region of technology $j \in 1, \ldots, k$ is given by*

$$\left([0, t_{j*}^D] \cup [t_{*j}^D, t^{lim}]\right) \cap [t_{*j}^C, t_{j*}^C],$$

where $t_{*j}^C = \max_{i<j} t_{ij}^{crit}, t_{j*}^C = \min_{i<j} t_{ji}^{crit}, t_{j*}^D = \min_{i>j} t_{ji}^{crit},\ t_{*j}^D = \max_{i>j} t_{ij}^{crit}$.

Note that by the preceding result, the inducibility region (if it exists) of some technology j consists of one or two closed intervals. We now turn our attention to the conditions for inducibility of the cleanest available technology k.

Proposition 2 *Technology k is inducible iff $t_{*k}^C \le t_{k*}^C$ and $t_{*k}^C < t^{lim}$.*

To visualize this result observe that in Example 2 $t_{*3}^C = \max\{0.884, 1.986\} = 1.986 \ge t_{3*}^C = \min\{1.790, 2.244\} = 1.790$, and so technology 3 is not inducible.

We note that in order to check the condition for inducibility in the previous result, one has to solve k non-linear equations $\Delta_{ik}(t) = 0$ for $i = 1, \ldots, k-1$. Moreover, even if the condition in Proposition 2 holds but the interval $[t^C_{*k}, t^C_{k*}]$ is small, technology k may not be practically inducible due to uncertainties about model parameters. Corollary 1, however, extends directly to the $k-$technology case: the width of the inducibility region $t^C_{k*} - t^C_{*k}$ is non-decreasing in Θ_k suggesting that subsidizing the fixed cost of the cleanest technology may be a good strategy for expanding the inducibility region. The issue of subsidizing fixed costs is further considered in the next section.

4.3 The Equal Fixed Costs Case

In the previous section we saw that the cleanest technology k may not be inducible even when the necessary conditions of Corollary 2 hold. In this section we show that when the fixed costs of all technologies are the same, the conditions of Corollary 2 are necessary and sufficient for technology k to be inducible (in fact, these conditions can be stated in a simplified form). We have the following proposition.

Proposition 3 *Suppose* $\Theta_i = \Theta_k$ *for all* $i = 1, \ldots, k$ *and let* $D(p)$ *be any non-negative continuous non-increasing demand function. Let* $t_{ik} = (\xi_k - \xi_i)/(1/S_i - 1/S_k)$. *Then*

(i) $t^C_{*k} = \min\left\{t^{lim}, \max_{\{i=1,\ldots,k-1\}} t_{ik}\right\}$ *and* $t^C_{k*} = t^{lim}$

(ii) *Technology* k *is inducible iff* $t^C_{*k} < t^{lim}$

The preceding result ensures that if there is no difference in the fixed costs between the technologies, and technology k is preferred for some tax level t', then it is preferred for any $t \in [t', t^{lim})$. Thus the negative environmental effect at higher tax levels cannot arise. Further, the result is very general because the Proposition holds for any demand function. We also note that Assumption 2 ensures that $t^C_{*k} > 0$, and Proposition 3 implies that technology k is inducible whenever it is pairwise-inducible with respect to every other technology. Thus, conditions in Corollary 2 are necessary and sufficient in this case. Of course, the conditions in Proposition 3 are also much easier to check since all quantities can be easily computed in closed form.

To summarize, two factors determine whether the cleanest technology k is competitive with other available technologies: the installation cost Θ_k compared with the installation costs of other technologies and whether k's environmental efficiency S_k is sufficient to offset (possibly) higher operating cost ξ_k at some feasible taxation level t. The preceding result suggests that by subsidizing the installation costs of the new technology, the regulatory body can ignore the first factor and induce technology k whenever it is competitive on the operating cost-only basis. Thus, a joint environmental tax and fixed cost subsidy approach may make technology k inducible even when it is not inducible under the taxation-only approach. An additional advantage of the fixed cost subsidy is that inducibility can be checked and assured under very general demand functions. The question, however, still remains whether it is optimal for the regulator to provide the subsidy. This, and other aspects of the regulator's decisions, are discussed below.

5 Regulator's Problem: Technological Choice and Social Welfare

In this section we analyze the problem of the regulator that anticipates the firm's reaction to the tax level and thus sets this level strategically to maximize social welfare. Let $Q(t)$ represent the social welfare associated with the taxation level t. We consider the optimization problem:

$$Q^* = \max_{t \geq 0} Q(t) \equiv \max_{t \geq 0} Q_i(t)|_{i=i^*(t)}, \tag{11}$$

where $Q_i(t)$ is the social welfare given tax level t set by the regulator and technology choice i made by the firm, and $i^*(t)$ is the firm's optimal technology choice in response to tax level t as per Section 4. Let $t^{opt} = \arg\max Q(t)$ be the socially optimal tax rate.

Note that our primary interest is not in the optimal tax rate itself, but in the socially optimal technology choice it induces, $i^*(t^{opt})$. We are particularly interested in the double dividend cases, i.e., the situations where the socially optimal technology is the clean one (in the 2-technology case, or the cleanest one in the k-technology case). Note that in general double dividends may be hard to achieve because the interests of different stakeholder are conflicting: e.g., the firm prefers a zero tax at which it selects the dirtiest technology and pollutes the most; similarly, should the firm choose a clean technology, it would pollute less and hence tax revenue would decline, and so on.

To balance these conflicting interests in line with the previous works, e.g., Atasu et al. (2009), we assume that the social welfare function consists of the following four components:

Social Welfare = Tax Revenue - Environmental Impact + Firm's Profit + Consumer Surplus.

Given our model assumptions from Section 3 for a given tax level t set by the regulator and a given technology choice i made by the firm, the components of the social welfare function can be expressed as follows:

- **Tax Revenue** is obtained by multiplying the amount of pollutant emitted, $\frac{x_i^*(t)}{S_i}$ by tax rate t. That is, from (5):

$$\text{Tax Revenue} = t\frac{\bar{A}(\delta-1)}{S_i}(c+\xi_i+t/S_i)^{-\delta},$$

  where as before $\bar{A} = A(\delta-1)^{(\delta-1)}/\delta^\delta$;

- **Environmental Impact** is obtained by multiplying the amount of pollutant emitted by a coefficient $\epsilon \geq 0$ that translates firm's emissions into monetary units:

$$\text{Environmental Impact} = \epsilon\frac{x_i^*(t)}{S_i} = \epsilon\frac{\bar{A}(\delta-1)}{S_i}(c+\xi_i+t/S_i)^{-\delta}.$$

  The coefficient ϵ plays an important role in our model as it measures the degree of environmental concern/awareness of the regulator and society; it also has a physical meaning, specifying the degree of environmental hazard of the firm's physical production process and the associated pollutant. High ϵ means that the society receives significant welfare loss because of emissions. Intuitively, the higher the value of ϵ, the higher is the regulator's desire to motivate the choice of the clean technology.

- **Firm's Profit** is given by (6).

- **Consumer Surplus** is obtained as the area under the demand curve above the optimal price. By integrating the demand function from (3) and substituting the expression for the optimal price from (4) we obtain that

$$\text{Consumer Surplus} = \int_{p^*(t)}^{\infty} D(p)dp = \frac{A}{\delta-1}[p^*(t)]^{(1-\delta)} = \bar{A}\frac{\delta}{\delta-1}(c+\xi_i+\frac{t}{S_i})^{1-\delta}.$$

Therefore, for a given tax level t and technology choice i, the social welfare is given by

$$Q_i(t) = (t-\epsilon)\frac{\bar{A}(\delta-1)}{S_i}(c+\xi_i+t/S_i)^{-\delta} + \bar{A}\frac{2\delta-1}{\delta-1}(c+\xi_i+\frac{t}{S_i})^{1-\delta} - (K+\Theta_i) \qquad (12)$$

which is quasi-concave and has a unique maximum at

$$t_i^{opt} = \frac{(\delta-1)\epsilon - S_i(c+\xi_i)}{\delta}. \qquad (13)$$

Two interesting observations can be made. First, if ϵ is small enough, then $t_i^{opt} < 0$, implying that the regulator is best off by setting $t = 0$. In light of Assumption 2 the firm then chooses the *dirtiest* available technology 1 because it results in the highest profit. In other words, small values of ϵ will have no effect on motivating green technology choice. It is straightforward to verify that for taxation to have a possibility of motivating the choice of technology i, the regulator must weigh environmental impact at the level of at least $\epsilon \geq S_i(c+\xi_i)/(\delta-1)$. That is, unless the society becomes moderately concerned with environmental impact, taxation will not have an impact on technology choice.

Second, observe that t_i^{opt} is increasing in ϵ. This is intuitive – the regulator that is more concerned with environmental impact should naturally have a tendency to set higher taxation levels. Note, however, that the actual social welfare function $Q(t)$, specified in (11), may not be maximized at any of the t_i^{opt} values: those values may lie outside the intervals over which the corresponding technologies are selected (i.e., it may happen that $i^*(t_i^{opt}) \neq i$ for all i). To find the optimal tax rate we therefore proceed as follows.

Recall from Lemma 1 that the inducibility region for each technology consists of at most two disjoint intervals. Since for every t some technology must be induced, these regions partition the interval $[0, t^{lim}]$ into at most $2k-1$ subintervals (since for technology k the inducibility region consists of a single interval). Thus, there exist $M \leq 2k$ *breakpoints* $t^B_{(m)}$, $m = 1, \ldots, M$ designating the boundaries of these subintervals such that

$$t^B_{(1)} = 0, t^B_{(M)} = t^{lim}, \text{ and } t^B_{(m)} < t^B_{(m+1)} \text{ for } m = 1, \ldots, M-1.$$

Of course each of these breakpoints $t^B_{(m)}$ (for $1 < m < M$) is equal to a critical value t^{crit}_{ij} for some technologies i, j with i being the optimal technology for $t \in [t^B_{(m-1)}, t^B_{(m)}]$ (i.e., $i^*(t) = i$ in this interval) and j being optimal for $t \in [t^B_{(m)}, t^B_{(m+1)}]$.

Note that while the firm's profit is the same whether a given breakpoint is approached from the left or the right (this follows by the definition of the critical points), the same is not true for the optimal production quantity, and hence for emissions and other components of the welfare function $Q(t)$. Indeed, letting $x(t) \equiv x_{i^*(t)}(t)$ denote the optimal production quantity corresponding to tax rate t, we see that for $t^B_{(m)} = t^{crit}_{ij}$ the limit of $x(t)$ as $t \uparrow t^B_{(m)}$ from the left is $x_i(t^B_{(m)}) \equiv x_i(t^{crit}_{ij})$, while the limit of $x(t)$ as $t \downarrow t^B_{(m)}$ from the right is $x_j(t^B_{(m)}) \equiv x_j(t^{crit}_{ij})$, and from (5) we know that, in general, $x_i(t^{crit}_{ij}) \neq x_j(t^{crit}_{ij})$.

We will use the notation $t^{B-}_{(m)}$ and $t^{B+}_{(m)}$ to designate left and right limits at this breakpoint; i.e., $x(t^{B-}_{(m)}) = x_i(t^B_{(m)})$ and $x(t^{B+}_{(m)}) = x_j(t^B_{(m)})$. The revenue function $Q(t)$ is continuous on each interval $[t^{B+}_{(m)}, t^{B-}_{(m+1)}]$ since it coincides with some $Q_j(t)$ function on this interval, but will, in general, have discontinuities at each breakpoint. In fact, since the firm is indifferent between the two production quantities at the breakpoint because they earn the same profit, the welfare function is not well defined at the breakpoints. This has important practical implications since both the firm and the regulator are very sensitive to small perturbations of the tax rate around each breakpoint: the welfare changes abruptly as the tax rate is changed from just below the breakpoint to just above it. In practice we may assume that there exists some small value $\mu > 0$ such that $t^{B-}_{(m)} = t^B_{(m)} - \mu$ and $t^{B+}_{(m)} = t^B_{(m)} + \mu$. With this convention we will regard the right and left limits at each breakpoint as two different tax rates.

Let $j(m)$ designate the induced technology for $t \in [t^B_{(m)}, t^B_{(m+1)}]$. Since $Q(t) = Q_{j(m)}(t)$ on this interval, the maximum value within this interval occurs either at one of the endpoints or at $t^{opt}_{j(m)}$ defined by (13), if this value falls within the interval. We say that $t^{opt}_{j(m)}$ is *feasible* if it falls in the interior of the interval.

Let $M^f = \left\{ m \in \{1, \dots, M-1\} | t^{opt}_{j(m)} \text{ is feasible } \right\}$ (this set may be empty). The preceding discussion leads to the following result.

Proposition 4 *Define the following discrete set of tax rates:*

$$T = \left\{ t^{B+}_{(m)}, t^{B-}_{(m)} | m \in \{2, \dots, M-1\} - M^f \right\} \bigcup \left\{ t^{opt}_{j(m)} | m \in M^f \right\}.$$

Then the welfare-maximizing (socially optimal) tax rate t^{opt} can be found in T.

To summarize, the following algorithm can be used to compute the socially optimal tax rate:

1. Compute inducibility region for each technology using Lemma 1. The boundaries of these regions yield the set of breakpoints and their number M.

2. Use (13) to compute t_j^{opt} for each technology j and identify whether this value is feasible (by checking whether it falls within the inducibility region of j).

3. Form set T defined in Proposition 4 and evaluate $Q_{i^*(t)}(t)$ for each element of T. The element yielding the maximum value is the socially optimal tax rate t^{opt}.

The case of *double dividend* occurs when t^{opt} belongs to the inducibility region of the cleanest technology k. Observe that this does not necessarily imply that $t^{opt} = t_k^{opt}$ (unless the latter is feasible) – it is possible to have double dividend when t^{opt} is set to one of the endpoints of the inducibility region of k.

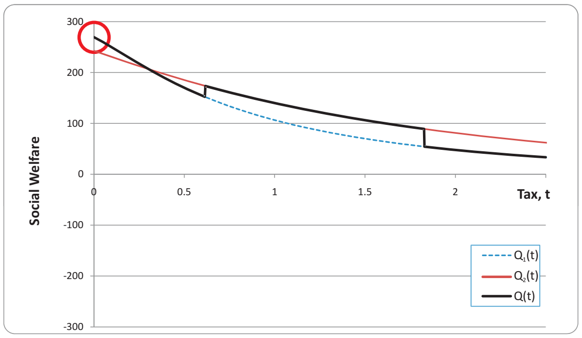
Note that, since by (13) t_k^{opt} is increasing and continuous in ϵ, it must be feasible for some value of the environmental concern parameter ϵ. However, if the regulator, extremely concerned with environmental impact, sets ϵ to a very large value, then t_k^{opt} may be above the upper boundary of the inducibility region of technology k, forcing t^{opt} to fall outside the inducibility region of technology k, i.e., $i^*(t^{opt}) \neq k$. The latter fact is interesting and counter-intuitive because it implies that:

Corollary 3 *An increase in the regulator's environmental concerns may motivate the firm to choose dirtier technology.*

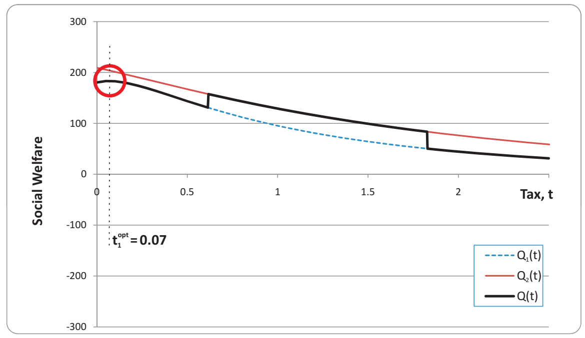
The location of socially optimal tax rate, the existence of the double dividend, as well as the "reverse" effect discussed above are easier to visualize on the example with two technologies.

Example 3 ***Social Welfare Maximization for the 2-Technology Case*** Continuing the Example 1, Figure 2 presents the plots of the total social welfare as a function of tax level for the cases with $\epsilon = 0.3, 0.6, 1.2, 3, 4.5, 6$ respectively.

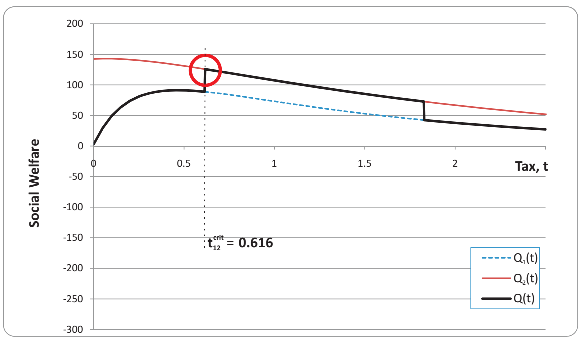
Case (a) illustrates the observation that whenever ϵ is too small, then the optimal tax will be zero (the bold line, denoting $Q(t)$ is highest at $t = 0$). In (b) with $\epsilon = 0.6$ the regulator is not particularly concerned with the environmental issues and therefore sets a low tax level $t^{opt} = t_1^{opt} = 0.07$ and by doing so motivates the firm to choose the dirty technology 1. In (c) through (e) the regulator is moderately concerned with the environmental issues ($\epsilon = 1.2, 3, 4.5$) and therefore sets tax levels that motivate the firm to choose the clean technology. These are the cases of double dividend. In (f) the regulator is extremely concerned with the



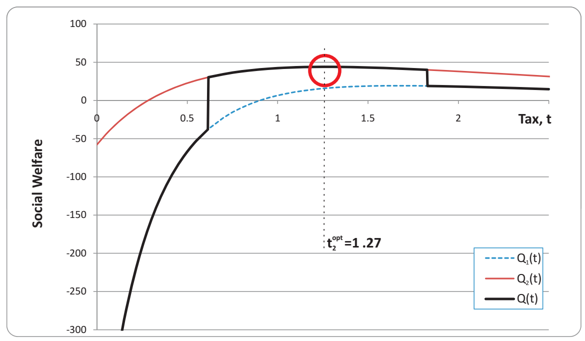
(a), $\epsilon = 0.3, t^{opt} = 0, i^* = 1$



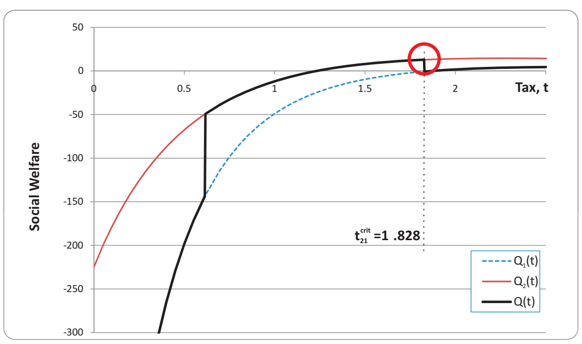
(b), $\epsilon = 0.6, t^{opt} = t_1^{opt} = 0.07, i^* = 1$



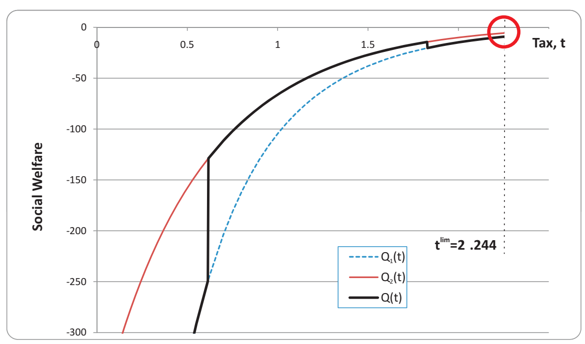
(c), $\epsilon = 1.2, t^{opt} = t_{12}^{crit} = 0.616, i^* = 2$ – double dividend



(d), $\epsilon = 3, t^{opt} = t_2^{opt} = 1.27, i^* = 2$ – double dividend



(e), $\epsilon = 4.5, t^{opt} = t_{21}^{crit} = 1.828, i^* = 2$ – double dividend



(f), $\epsilon = 6, t^{opt} = t^{lim} = 2.244, i^* = 1$

Figure 2: Social welfare as a function of tax rate for the cases with $\epsilon = 0.3, 0.6, 1.2, 3, 4.5, 6$ respectively.

Interval of ϵ values	0-0.5	0.5 - 0.8	0.8-2	2-3.9	3.9 - 5.2	> 5.2
Socially optimal tax level	0	$t^{opt} = t_1^{opt}$	$t^{opt} = t_{12}^{crit}$	$t^{opt} = t_2^{opt}$	$t^{opt} = t_{23}^{crit}$	$t^{opt} = t_1^{lim}$
Technology choice	dirty	dirty	clean/boundary	clean	clean/boundary	dirty

Table 1: The optimal tax level and technology choice for different values of ϵ.

environmental issues ($\epsilon = 6$), but doing so forces the tax level so high, that in response to it the firm is choosing not the clean technology, but the dirty one (Corollary 3).

Beyond the cases presented on Figure 2, the relationship between the socially optimal tax level and technology choice for different values of ϵ is summarized in Table 1. In addition to what we already saw above, notice that for a significant range of environmental concerns values, the optimal taxation level is at the boundary between the two technology choices. Since from Corollary 1 those boundaries shift if a fixed costs change, providing a fixed cost subsidy could be potentially beneficial as we discuss next.

6 The Role of Subsidies

As we argued in Section 4.3, providing a subsidy for the installation cost of clean technology removes the negative environmental effect, and as a result once a certain tax-level threshold is crossed, the firm will always chose the clean technology. However, having the opportunity to provide such subsidy, would the regulator chose to do so; i.e., does providing a subsidy increase the social welfare? In this subsection we answer that question by studying the effects of subsidy on the socially optimal tax level, the technology choice it motivates, and the resulting effect on the social welfare.

For simplicity we consider the case with two technologies, where technology 1 is a null technology with $\Theta_1 = \xi_1 = 0$ and $S_1 = 1$, and technology 2 is described by $\Theta_2, \xi_2 > 0$ and $S_2 > 1$; all results can be extended to the $k-$technology case. To be consistent with earlier results in Section 4.3 we only consider the case of full subsidy. i.e., $Subsidy = \Theta_2$, to make the installation costs of both technologies equal. More general mechanisms of both subsidizing only a part of the installation costs ($Subsidy < \Theta_2$) or subsidizing more than the installation costs ($Subsidy > \Theta_2$) could also be of interest, but will not be considered in the current paper. We also do not consider variable cost subsidies: in practice those are

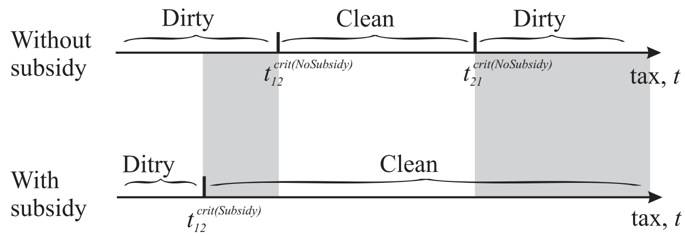


Figure 3: The effect of subsidy on critical taxation levels.

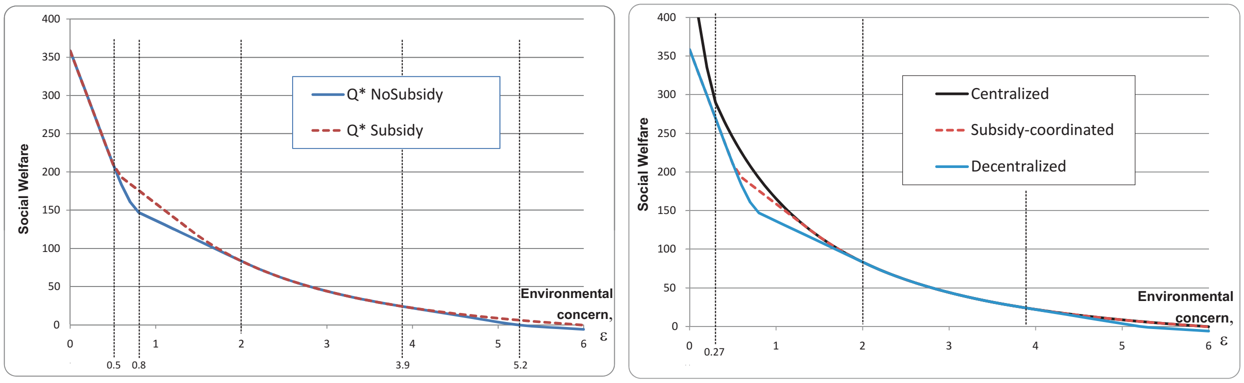
frequently illegal because of trade agreements.

The subsidy has two fundamentally different effects. For a fixed technology choice i, the social welfare function (12) and its maximizer (13) are independent of subsidy for all t. This happens because if the firm chooses technology 1 then there is no subsidy at all, and if it chooses technology 2 then the regulator's tax revenue is decreased by Θ_2 and the firm's profit is increased by Θ_2; hence there is no net effect on total social welfare. However, the firm's optimal technology choice, $i^*(t)$, is changing with subsidy because critical taxation levels change as per Corollary 1.

Specifically, from Proposition 3, with full subsidy there is only one taxation level, $t_{12}^{crit(Subsidy)}$ at which the firm switches from dirty technology 1 to clean technology 2. From Corollary 1, $t_{12}^{crit(Subsidy)} \leq t_{12}^{crit(NoSubsidy)}$. Thus, for $t \in T^0 \equiv [0, t_{12}^{crit(Subsidy)}] \bigcup [t_{12}^{crit(NoSubsidy)}, t_{21}^{crit(NoSubsidy)}]$, the firm's technology choice is identical with or without subsidy. On the other hand, for $t \in T^S \equiv [0, t^{lim}] - T^0$, the firm's technology choice swtiches from "dirty" (without subsidy) to "clean" (with subsidy). The set T^S is is depicted by the shaded areas on Figure 3. Therefore:

Proposition 5 *Providing subsidy improves technology choice:* $i^*_{Subsidy}(t) \geq i^*_{No\ Subsidy}(t)$ *for all* t, ϵ.

Note that the set T^S consists of both the "low" rates at which without subsidy the clean technology was not selected yet, and the "high" tax rates at which without the subsidy the firm switched back from clean to dirty because of the negative environmental effect. Thus, since by (13) t_2^{opt} is increasing in ϵ, there exists some ϵ' such that for $\epsilon > \epsilon'$ $t_2^{opt} \in T^S$, i.e., if the subsidy is provided, then the clean technology is induced at the socially optimal tax rate. Therefore in contrast with Corollary 3 we have that:



(a) (b)

Figure 4: Social welfare with and without subsidy as a function of environmental concern parameter ϵ (a), same compared with the first best solution, (b).

Corollary 4 *With full subsidy, once environmental concerns are high enough, the double dividend is achieved: the tax level that maximizes the social welfare motivates the firm to choose the clean technology.*

To establish the impact of providing full subsidy on the optimal social welfare and the optimal tax rate, observe from (12) that when ϵ is high enough, then for all t $Q_2(t) \geq Q_1(t)$. This follows because $Q_i(t)$ is a linear decreasing function of ϵ and the coefficient of ϵ is smaller for $i = 2$ because $S_2 > S_1$ and $\xi_2 > \xi_1$. Thus, if for a given ϵ the optimal tax rate falls into T^0, then the optimal welfare is unchanged. Otherwise, the technology choice improves and as a result the welfare increases, leading to the following result (see Appendix for the proof):

Proposition 6 *If environmental concerns are high enough, then $Q^*_{Subsidy} \geq Q^*_{NoSubsidy}$. That is, providing subsidy increases social welfare.*

Interestingly, the effect of subsidy on the socially optimal tax rate is non-monotone. Intuitively, by providing the subsidy the regulator should be able to decrease the tax rate. And indeed, the left critical taxation level is decreased (i.e., $t_{12}^{crit(Subsidy)} < t_{12}^{crit(NoSubsidy)}$); see Figure 3. However, with subsidy the upper critical level $t_{21}^{crit(NoSubsidy)}$ is removed. Therefore:

Corollary 5 *Providing full subsidy could increase or decrease the socially optimal tax rate.*

The three results are illustrated in Figure 4 (a) for the same set of parameters as in the preceding example. For $\epsilon < 0.5$ from Table 1 the optimal tax rate is zero and subsidy is not applicable (the firm always chooses the dirty technology). For $\epsilon \in (0.5, 0.8)$ with subsidy the firm is choosing the clean technology (Proposition 5) and thus the social welfare increases (Proposition 6) as a result of providing subsidy. The welfare also increases for $\epsilon \in (0.8, 2.0)$ but for the different reason. Over that range from Table 1 the optimal tax rate was on the left boundary of the clean technology's Region II. With subsidy that boundary shifts left (Figure 3), i.e., the optimal tax rate decreases (Corollary 5) leading to an increase in the social welfare. For $\epsilon \in (2.0, 3.9)$ the optimal tax rate is t_2^{opt} which from (13) is independent of subsidy, hence providing a subsidy has no effect on the social welfare. For $\epsilon \in (3.9, 5.2)$ t_2^{opt} is above the right boundary of the Region II and hence without subsidy from Table 1 the tax rate is set at the boundary. With subsidy the boundary is removed (Figure 3) and hence the optimal tax rate increases to t_2^{opt} (Corollary 5). Finally, because subsidy removes the negative environmental effect, for $\epsilon > 5.2$ the firm continues to choose the clean technology (Proposition 5), the optimal rate therefore is t_2^{opt}, and the welfare increases as well.

To summarize, our results show that providing a full subsidy for the installation cost of clean technology (in coordination with emissions taxes) is, generally, a very good idea. Doing so increases the social welfare and improves the technology choice, as well as removes an undesirable case when an increase in the environmental concerns forces the firm to choose the dirty technology. Interestingly, however, providing a subsidy may increase the optimal tax rate.

6.1 Fully Coordinated, Subsidy-Coordinated and Decentralized Solutions

The main model considered in the current paper assumes that the regulator does not have the direct control over either the price or the production quantity – these decisions are made independently by the firm in response to the tax rates set by the regulator; we refer to the resulting solution as "decentralized". In the previous section we considered the case where fixed costs subsidies are offered to offset the acquisition costs of cleaner technologies, we refer to the resulting solution as "subsidy-coordinated". It was shown in Proposition 6 that, with respect to optimizing social welfare, the subsidy-coordinated solution is superior to the decentralized one. In the current section we introduce a third type of solution –

"fully coordinated" or "centralized" (sometimes also known as the "first best"), where the regulator has direct control over tax rates, prices and production quantity. Clearly, this type of solution will achieve the highest level of social welfare. The goal of the current section is to analyze the fully coordinated solution and compare it to the other two solution types.

To compute the fully coordinated solution we proceed as in Section 5 except that instead of the firm's optimal quantity and price are no longer obtained from the results in Section 4. Instead we substitute a decision variable for price, p, resulting in the optimal production quantity of $D(p) = Ap^{-\delta}$. For a given ϵ, the central planner must pick the values of (p, t, i) to maximize:

$$\begin{aligned} Q_{i \text{ full coord}}(p,t) &= \text{Tax Rev - Env Impact + Firm's Profit + Consumer Surplus} \\ &= (t-\epsilon)\frac{D(p)}{S_i} + \left(p - (c + \xi_i + \frac{t}{S_i})\right) D(p) - \Theta_i - K + \int_p^\infty D(p)dp \\ &= \left(p - (c + \xi_i + \frac{\epsilon}{S_i})\right) D(p) - \Theta_i - K + \int_p^\infty D(p)dp. \end{aligned} \tag{14}$$

Observe that t cancels out, which is not surprising – taxation plays no role when the central planner that controls both firm's and consumer's surpluses directly. It is also not difficult to verify that the above function is unimodular in p, and that the optimal price satisfies the natural property that one should expect to hold for the central planner: price = marginal (societal) cost, i.e.:

$$p^*_{\text{fully coord}} = c + \xi_i + \frac{\epsilon}{S_i} \tag{15}$$

Interestingly, if in the leader-follower problem (i.e. the decentralized solution case) the regulator finds it optimal to set the tax rate at t_i^{opt} as given by (13), then it is not difficult to see that substituting the tax rate from (13) into the expression for the optimal price in (4), we obtain the expression above for the optimal price. This immediately leads to the following result:

Proposition 7 *If for some technology i , the value of t_i^{opt} falls into the region over which the firm selects technology i, i.e. $t_i^{opt} \in \left([0, t^D_{j*}] \cup [t^D_{*j}, t^{lim}]\right) \cap [t^C_{*j}, t^C_{j*}]$ using the notation of Lemma 1, then centralized, subsidy-coordinated and decentralized solutions are identical with respect to technology choice, optimal prices and production quantities.*

Combining this result with Proposition 5 we immediately obtain the following corollary:

Corollary 6 *If ϵ is sufficiently high, then subsidy-coordinated and centralized solutions are identical.*

Figure 4 (b) illustrates these results. As we discussed earlier, t_2^{opt} becomes the regulator's optimal choice once ϵ exceeds 2. When subsidy is provided, t_2^{opt} is optimal for all $\epsilon \geq 2$ – hence the centralized and subsidy-coordinated solutions are identical on that interval. When subsidy is not provided, t_2^{opt} stops being the optimal choice when ϵ increases above 3.9; see Table 1. Thus centralized and decentralized solutions coincide for $\epsilon \in [2, 3.9]$ but the centralized and subsidy-coordinated solutions achieve higher welfare than the decentralized solution when $\epsilon > 3.9$.

Overall, these results are certainly encouraging. They, once again, suggest that providing fixed cost subsidy may be a good strategy: it not only improves the social welfare, but in fact brings it up to the highest possible level if the society is sufficiently concerned with the environmental impact.

7 Conclusions and Future Research

We discuss the ability and limitations of using environmental taxes or pollution fines to motivate firms to adopt innovative and "green" emissions-reducing technologies. Within a Stackelberg game model we first consider the firm's technology choice, pricing, and production decisions in response to a tax level set by the regulator, and then consider how the regulator should set the tax level strategically in order to maximize social welfare.

We show that while environmental taxation can be effective in motivating the adoption of clean and green emissions-reducing technology, it has to be used with caution since overly high tax levels can actually demotivate the choice of clean technologies due to the negative environmental effect that we show to exist. Moreover, the ability of taxation to motivate the choice of clean technologies may be limited if the firm has many technologies to choose from: in that case, the cleanest technology may never be induced; generally for a technology to be inducible with taxation, its environmental efficiency must be sufficiently high relative to its operating and capital costs. Our results also indicate how subsidies may be used: if the capital cost of cleaner technologies is subsidized, then negative environmental effect disappears and taxation becomes very efficient.

From the regulator perspective, we show that the economic and environmental objectives are not necessarily in conflict: The tax level that maximizes social welfare may simultaneously motivate the choice of clean technology, resulting in the case of double dividend. Such cases happen when the regulator is moderately concerned with environmental impacts. When environmental concerns are low, intuitively, it is socially optimal for the regulator to set a low tax rate and motivate the choice of dirty technology; furthermore, when the environmental concerns are too low, we show that taxation should not be used at all (the optimal tax rate is zero). What is interesting and somewhat counterintuitive is that when the environmental concerns are very high, the optimal tax rate may also motivate the choice of dirty technology. That happens because of the negative environmental effect: high environmental concerns lead to high tax rates, which in turn lead to the choice of dirty technology.

Since the latter negative effect is removed with subsidy, providing one is overall beneficial. Indeed, we show that by providing a fixed cost subsidy and optimally taxing emissions the regulator improves the technology choice and increases the social welfare. Note, however, that subsidy may increase the optimal tax rate. By comparing the centralized, subsidy-coordinated and decentralized solutions, we show that, provided the societal environmental concerns are high enough, the subsidy-coordinated and the decentralized solutions coincide – i.e., the subsidy-coordinated solution achieves the highest possible level of social welfare.

We close the paper with several further comments regarding our model. First, while our primary interest has been in the interaction between a government regulator and a polluting firm, more general examples where some regulatory body is using tax-like fines to motivate desirable behavior of the "violator," yet is very cognizant of the need to maintain the revenue stream resulting from these violations, are quite common outside the field of environmental analysis. For example, illegal parking can be viewed as a form of "pollution" – it is an undesirable by-product of doing business and could be reduced or eliminated by the choice of a more "expensive" technology (paying for legal parking or using public transit). Parking fines add up to millions of dollars for most municipalities and constitute an important source of revenue: Chung (2007) reports that the city of Toronto nets a "curbside" revenue of CAD 80 million per year, 1.5 million of which comes from its three most-ticketed "clients": FedEx, UPS and Purolator. As Chung notes: *"They have no intention of parking legally. The city has no intention of halting the ticketing."* Indeed, while municipalities could, presumably, set the fines at prohibitive levels to induce near-perfect compliance, the resulting loss of revenue is not an attractive proposition. Instead, the fines seem to be set at the levels that ensure moderate non-compliance and the resulting revenue stream (explicit targets for the latter are often included in city budgets). Similar examples can be found with respect to many other organizations.

Second, observe that the role of the "regulator" may be played by a non-government agent. For example, in some cases the headquarters of a multinational firm may be interested in inducing certain technology choice behavior from its subsidiaries through internal taxation-like transfer pricing mechanisms.

Finally, note that the whole concept of "technology change" can be also viewed rather broadly. For example, relocating a business to another geographical region, where, renewable energy is more widespread, could also be viewed as a technology change[¶]. The move involves a fixed cost, additional shipping fees may be involved and thus the variable cost increases, but the energy related taxes could decrease.

In terms of future work, the immediate extension that we address in the follow-up paper Krass et al. (2010) is the oligopolistic case with multiple heterogenous firms competing in a Cournot fashion. Other extensions could explicitly incorporate uncertainty and asymmetry of information, both in demand and in technologies' cost or environmental performance, as well as to explicitly consider how motivating the adoption of the clean technologies propagates down the "innovation chain," motivating the firms that develop these technologies to innovate. Case-based and empirical research is also of interest.

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A Proofs

Proof of Proposition 1

First observe that for A sufficiently large, $t^{\lim} > t^{max}$. This follows because by definition $t^{\lim}$ is increasing in A, while from (8) t^{max} is independent of A.

Let $\Theta = \Theta_2 - \Theta_1$ be the difference in fixed costs of technologies 1 and 2. From (7) it is easy to see that

$$\Delta(t) = f(t) - \Theta,$$

where $f(t)$ is independent of Θ_1, Θ_2. Moreover, under assumption $\xi_1 < \xi_2$, $f(t)$ is a quasi-concave function increasing for $t \in (0, t^{max}]$. Thus, by selecting new values Θ_1', Θ_2' of fixed cost parameters so that $\Theta' = \Theta_2' - \Theta_1' = f(t^{max})$, we ensure that $\Delta(t^{max}) = 0$ and $\Delta(t) < 0$ for all $t > 0,\ t \neq t^{max}$.

Now, choose $\epsilon > 0$ and suppose the fixed cost parameters are adjusted so that $\Theta'(\epsilon) = \Theta' - \epsilon$. Then $\Delta(t^{max}) = \epsilon$, and $\Delta(t)$ has exactly two roots on $(0, \infty)$ provided ϵ is sufficiently small. Let $t_2^{crit}(\epsilon)$ be the larger root. Note that $t_2^{crit}(\epsilon) > t^{max}$ for $\epsilon > 0$ and is a continuous function of ϵ. Thus, if the market capacity assumption $t^{max} < t^{lim}$ holds there must exist $\epsilon > 0$ such that $t^{max} < t_2^{crit}(\epsilon) < t^{lim}$, implying that Region III, which equals to $t \in (t_2^{crit}(\epsilon), t^{lim})$, is non-empty. To summarize, we have shown that under the assumptions of the Proposition there always exist values of fixed cost parameters Θ_1, Θ_2 for which Region III is non-empty.

$\blacksquare$

Proof of Proposition 2

Technology k is inducible iff $t_{*k}^C \leq t_{k*}^C$ and $t_{*k}^C < t^{lim}$. From the previous lemma and since k is the cleanest technology (i.e., has the highest index), it follows that the inducibility region of k is given by $[t_{*k}^C, t_{k*}^C]$. This regions is non-empty whenever the conditions in the hypothesis hold. Q.E.D.

Proof of Proposition 3

Consider technologies k and $i \neq k$. Recall that, by assumption, $S_k > S_i$. Note that for arbitrary $y_k \leq y_i$, we have

$$D(p)(p - y_i) \leq D(p)(p - y_k)$$

for any $p \geq 0$. Taking maximum over $p \geq 0$ of both sides above, we see that $Pr_k(t) \geq Pr_i(t)$. Thus, $\Delta_{ki}(t) \geq 0$ if and only if

$$\xi_k + t/S_k \leq \xi_i + t/S_i,$$

which is equivalent to $t \geq t_{ik}$ where the latter quantity is defined in the statement of the Proposition. This implies that $t_{ik}^{crit} = \min\{t^{lim}, t_{ik}\}$ and $t_{ki}^{crit} = t^{lim}$, which, together with Lemma 2, imply part (i) of the Proposition. Part (ii) now follows immediately by Proposition 2. ■

Proof of Proposition 6.

Consider an arbitrary ϵ such that for all t $Q_2(t) \geq Q_1(t)$. Let $t^{opt}_{NoSubsidy}$ be the optimal tax rate for that ϵ without subsidy; let $Q^*_{NoSubsidy}$ be the corresponding optimal social welfare, and similarly for the variables with subscript $Subsidy$.

If $t^{opt} \in T^0$ then $t^{opt}_{NoSubsidy} = t^{opt}_{Subsidy}$ and so $Q^*_{NoSubsidy} = Q^*_{Subsidy}$. If $t^{opt}_{NoSubsidy} \in T^S$ and $i^*_{NoSubsidy} = 1$ then: $Q^*_{Subsidy} \geq max_{t \in T^S} Q_2(t) \geq Q_2(t^{opt}_{NoSubsidy}) \geq Q_1(t^{opt}_{NoSubsidy}) = Q^*_{NoSubsidy}$. If $t^{opt}_{NoSubsidy} \in T^S$ and $i^*_{NoSubsidy} = 2$ then either $t^{opt}_{NoSubsidy} = t^{crit+}_{12}$ or $t^{opt}_{NoSubsidy} = t^{crit-}_{21}$. If the former is the case then because $Q_2(t)$ is unimodular, $Q_2(t)$ is decreasing at $t^{opt}_{NoSubsidy}$; hence $Q^*_{Subsidy} \geq max_{t \in T^S} Q_2(t) \geq Q_2(t^{crit-}_{12}) \geq Q_2(t^{crit+}_{12}) = Q^*_{NoSubsidy}$ for some value $t^{crit-}_{12} \in T^S$ just to the left of t^{crit}_{12}. If the latter is the case then $Q_2(t)$ is increasing at t^{crit-}_{21}, implying that $t^{opt}_2 \geq t^{crit}_{21}$. Thus $Q^*_{Subsidy} \geq max_{t \in T^S} Q_2(t) = Q_2(t^{opt}_2) \geq Q_2(t^{crit-}_{21}) \geq Q^*_{NoSubsidy}$. ■

B The Linear Demand Function Case

In this appendix we demonstrate that the non-monotonicity in the firm's response to the environmental tax also holds for the case when the firm's demand function is linear (as opposed to the exponential demand we considered earlier).

Let $D(p) = a - bp$. Then given technology choice, i, and tax level, t, the firm's profit function is

$$Pr_i(p,t) = -bp^2 + (a + b(c + \xi_i + t/S_i))p - (c + \xi_i + t/S_i)a - (K + \Theta_i) \tag{16}$$

which is maximized at

$$p_i^*(t) = \frac{a + b(c + \xi_i + t/S_i)}{2b}, \tag{17}$$

with the associated profit level, $Pr_i(t)$, obtained by substituting (17) into (16) as follows:

$$Pr_i(t) \equiv Pr_i(p_i^*(t), t) = \frac{(a + b(c + \xi_i + t/S_i))^2}{4b} - (c + \xi_i + t/S_i)a - (K + \Theta_i). \tag{18}$$

It is easy to show that $Pr_i(t) \geq 0$ iff $0 < t \leq t_i^{lim}$, where

$$t_i^{lim} = S_i(\frac{a}{b} - c - \xi_i). \tag{19}$$

We assume that $t_i^{lim} > 0$ for all i, since otherwise technology i would never be chosen. Further for all technologies $i = 1, 2, ...k$ to be feasible, taxation level, t, must be bounded by t^{lim}, where

$$t^{lim} = \min_{i \in \{1,...k\}} t_i^{lim}. \tag{20}$$

As before, define

$$\Delta(t) = Pr_2(t) - Pr_1(t), \tag{21}$$

meaning that if $\Delta(t) > 0$ then the firm would select a cleaner technology 2, and otherwise it would select a dirtier technology 1. By definition cleaner technology 2 is more expensive; in particular, we assume $\xi_2 \geq \xi_1$ and $\Theta_2 \geq \Theta_1$ with at least one inequality being strict.

Let $\alpha_i = b/4S_i^2$, $\beta_i \equiv t_i^{lim} = S_i(a/b - c - \xi_i)$ and $\gamma_i = K + \Theta_i$. Note that with our assumptions $\alpha_1 > \alpha_2$ and $\gamma_1 < \gamma_2$. With this notation, the equation $\Delta(t) = 0$ can be rewritten as:

$$\alpha_2(t - \beta_2)^2 - \gamma_2 - \alpha_1(t - \beta_1)^2 + \gamma_1 = 0, \tag{22}$$

and therefore if

$$\gamma_2 - \gamma_1 < \frac{\alpha_1\alpha_2(\beta_1 - \beta_2)^2}{\alpha_1 - \alpha_2}, \tag{23}$$

then the equation (22) has two roots (there is only one root if (23) holds as an equality, and there are no roots if the opposite strict inequality holds.) Simple algebra shows that these two roots are:

$$t_{1,2}^{crit} = \frac{\alpha_1\beta_1 - \alpha_2\beta_2 \pm \sqrt{\alpha_1\alpha_2(\beta_1 - \beta_2)^2 - (\alpha_1 - \alpha_2)(\gamma_2 - \gamma_2)}}{\alpha_1 - \alpha_2}$$

As before, to conclude we show that there exist a set of parameters for which these two roots are feasible: that is the roots themselves and the profits obtained by the firm at these roots are non-negative.

Proposition 8 *If $\beta_2 > \beta_1$ and $\alpha_1\beta_1 > \alpha_2\beta_2$, then there exist γ_1 and γ_2 such that $0 < t_1^{crit} < t_2^{crit} < t^{lim}$.*

First, as we have just showed, roots $t_1^{crit} < t_2^{crit}$ exist if (23) holds. These roots are positive if $\Delta(0) < 0$ and $\Delta'(0) > 0$. The second inequality holds if $\alpha_1\beta_1 > \alpha_2\beta_2$ which is assumed by the condition of the Proposition. The first inequality holds when $\alpha_2\beta_2^2 - \alpha_1\beta_1^2 < \gamma_2 - \gamma_1$, which does not contradict the existing conditions. Lastly, with the assumed conditions, it can be shown that $t^{lim} = \beta_2 - \sqrt{\gamma_2/\alpha_2}$ and thus choosing γ_2 such that $\gamma_2 < \alpha_2(t_2^{crit} - \beta_2)^2$ together with the above conditions ensures that both roots exist, are positive and do not exceed t^{lim}, i.e., the firm attains a positive profit in each of the three critical regions. ■

Note that only a single root and hence only two regions exists if $\Theta_1 = \Theta_2$; this follows from Proposition 3 which holds for a general demand function.