Revenue and Cost Management for Remanufactured Products

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June 2009
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Abstract

This paper studies joint pricing and remanufacturing strategy of a firm that decides to offer both new and remanufactured versions of its product in the market. In making this decision, the firm balances the additional revenue from selling remanufactured products, the demand cannibalization of new product sales, and the cost of remanufacturing. We present the model of the firm’s decisions and complement it with a study of consumer behavior regarding the choice between new and remanufactured products. Our model does not rely on knowing consumers’ willingness to pay (WTP), but rather on knowing the fraction of consumers who switch from the new to the remanufactured product. In the behavioral study, we find that the switching fraction has an inverted U-shape, and thus, the underlying consumer behavior cannot be modeled using the WTP approach.

Combining theoretical and behavioral results, we find that the firm in most cases follows one of three strategies – remanufacture nothing, remanufacture everything, or resell falsely returned products. We characterize the optimality conditions for each strategy and relate them to the underlying product and market parameters (consumer behavior, price, demand elasticity, and others).

We find that the firm that incorporates the inverted-U-shaped consumer behavior in its pricing and remanufacturing strategy effectively triples the incremental profit it receives from remanufacturing, as compared with when the firm makes its decisions based on the WTP only. This happens because with the inverted-U, the firm remanufactures under broader conditions, charges a much lower price, and remanufactures more units. We also find that the size of the low-price market segment plays an important role because the firm reacts to it differently than the WTP-based logic would suggest.
1 Introduction

In many cases, firms that consider remanufacturing some of their returned products are rightly concerned that these remanufactured products offered at lower prices will cannibalize the sales of the higher-margin new products. As a result, many if not most firms do not offer remanufactured products together with new ones. For example, Dell has a separate Web site (www.delloutlet.com) where it sells its refurbished computers and accessories. At the same time, offering remanufactured products together with new ones allows firms to better segment their demand and capture sales to the “low-end” customers who will not purchase the new product. Therefore, even if some sales of the new product would be cannibalized, offering both new and remanufactured products could be profitable for the firm overall, provided that the price and the quantity of remanufactured products are selected properly.

The goal of this paper is to analyze the operations and pricing strategy for a firm that is considering putting remanufactured products on the market together with new products. We begin by providing the short case study that motivated our work.

1.1 Mini Case Study: R

This work was motivated by discussions with the management of company R (name disguised for confidentiality purposes) – a major wireless carrier in North America. R sells telephones, PDAs, and data devices as well as mobile voice and data plans that accompany these devices. R has an extensive network of retail outlets; it also sells its products and services online. In particular, in the spring of 2007, its Web site offered both new and refurbished versions of a number of devices. For example, a popular PDA was offered at $399 new and $139 refurbished. Interestingly, with the default sort by price (ascending), R’s customers saw the refurbished product well before seeing the new one.

With such a big price difference, the remanufactured product clearly cannibalized some sales of the new one. Yet R had both products available. This led us to study its marketing and operations with respect to remanufacturing. As a result, we learned that

*In the electronics industry “refurbished” and “remanufactured” are used as synonyms and we also use these terms interchangeably throughout the paper.
there are several key facts that differ from what is typically assumed in the literature.

First, R’s remanufacturing operations had no effect on the pricing, procurement or other decisions about the new products. R’s management noted that the pricing and procurement of new products is generally affected by such factors as the overall composition of R’s product line, competitive moves, seasonal and regional promotions, and so on, the total of which bore a far greater weight on R’s strategy than remanufacturing activities. They further noted that consumers have a negative perception of refurbished products, and pricing refurbished products too low can be interpreted by some consumers as a further negative signal about a product quality or popularity. At the same time, there exist other more price-sensitive and less quality-sensitive consumers who will “upgrade” to a refurbished version of an expensive product from a different lower-level new product. Thus R’s goal in refurbished product pricing was to achieve a delicate balance with customer perceptions.

Pricing of refurbished products, however, was not R’s primary concern; rather, it was the quantity of remanufactured products R wants to put on the market. Underlying this was a decision about the number of remanufacturable cores† to acquire. The interplay of these two decisions is explained by the contractual arrangement of R’s remanufacturing operation, described below.

R’s remanufacturing activity is done by a third party (partly owned by R). R specifies both the number of cores to acquire and test (for which it pays a flat rate per core) and the number of units to refurbish. Refurbishing is priced on a tier-based scale. Refurbishing a tier-1 core costs R about three times the cost of testing, and tier-1 cores correspond to about 60% of the number of cores procured. Refurbishing a tier-2 core costs about nine times that of testing. Together, tier-1 and tier-2 cores account for more than 90% of all cores. There also exist tier-3 and tier-4 cores, the remanufacturing of which is much more costly. Interestingly, R’s management was unable to say what, exactly, it would

†A “core” is a returned product that cannot be immediately resold as new – possibly opened, used, and/or defective product. By R’s standards, which it believes are used industry-wide, a device with less than 3 minutes of airtime (most devices have a built-in lifetime timer that records total use time since the device was made) can be sold as new. Devices with more than 3 minutes of airtime, even if they look new and are fully functional, can only be sold as refurbished.
cost to remanufacture tier-3 and -4 cores, an indication how infrequently this happens in practice.

The different tiers reflect the heterogeneity in the condition of the cores. Indeed, the number of procured cores corresponds to so called false failure returns – the returns of effectively new products that “have no verifiable functional defect” (Ferguson et al. 2006). For example, think of a consumer who bought a fancy PDA and then discovered that the buttons were a bit too small for his somewhat larger-than-average fingers. The box is opened, some of the packaging is destroyed, but otherwise the product is fully functioning. That would be a tier-1 core. Once all such cores are exhausted, the remaining cores have something broken, e.g., parts or software. But still, it is often possible to find cores that have a combination of working parts that can be assembled into a working product. For example, if one telephone has a broken keyboard, and another has a broken screen, then taking the screen from the first one and putting it into the second results in a working telephone. Such a device would correspond to tier-2. Swapping the screens involves some labor and extra testing, and thus tier-2 rebuilds are more expensive. The remaining device (with keyboard and screen both broken) is much more expensive to refurbish, since new parts have to be bought: that would be tier-3 or -4.

The key implication of this tiered cost structure is that the marginal cost of remanufacturing could be decreased by acquiring more cores. This is indeed often done by R’s management, which sometimes deliberately orders more cores than it plans to remanufacture.

In sum, with respect to remanufacturing a particular product, R is making three decisions: how many remanufacturable cores to acquire, how many products to remanufacture (of these acquired cores), and how to price them relative to the given price of the new product.

Beyond R, the features of the remanufacturing problem we described in this mini case study seem to be applicable rather generally. One could reasonably expect that in their remanufacturing activities, firms in many cases deal with at least some confirmation of the tiered cost structure, collection and remanufacturing decisions, no desire to “mess” with the new product’s price, and consumers’ judging quality from price.

These characteristics of R’s problem, however, do not fit the existing models of reman-
ufacturing well. On the revenue side, the interplay between quality perception and price (to our knowledge) has not been studied within the remanufacturing domain. On the cost side, tiered structure cost adds complexities beyond what most models assume (see Section 2 for literature review). R’s problem calls for the development of a new model that integrates revenue and cost management components, and this paper presents such a model.

1.2 Contributions of the paper

This paper presents a new stylized model of how a firm makes remanufacturing decisions, one that more accurately describes the revenue and cost components of remanufacturing, and is supported by the study of consumer behavior with respect to the choice of new versus remanufactured products. The approach in this paper is distinguished from previous research (see Section 2) on both modeling and behavioral/empirical basis.

From the modeling perspective, our key contributions are the following. First, we separate the decisions about the number of cores to acquire and the number of units to remanufacture, and consider a nonlinear cost of remanufacturing where the marginal cost of remanufacturing a unit increases in the number of units to remanufacture, but decreases in the number of available cores (reflecting the tiered structure in the case of R). Second, with respect to consumer behavior, we do not rely on knowing the distribution of consumer willingness to pay (WTP), but rather suggest a construct where the firm’s decisions are guided by estimating the fraction of consumers who, for a given price difference, would switch from the new to the remanufactured product. An implicit assumption behind any model that bases consumer choice on WTP is that “the cheaper, the better,” i.e., that a consumer is willing to purchase the product at any price below his or her WTP. With respect to remanufactured (consumer) products, however, this assumption is not consistent with observed behaviors. In particular, consumers may judge the quality of the product from its price. A very low price may signal that the product is of an inferior quality, thus decreasing the number of consumers willing to purchase it. Such a relationship between price and perceived quality is well-known in the marketing literature, for example, see Rao and Monroe (1989) or Völckner and Hofmann (2007). We also observe it in our behavioral study. Our model does not rely on knowing WTP and
is therefore capable of accommodating such complex and realistic consumer behavior. Further, estimating consumer behavior relative to the price and demand for the new product is arguably more robust: the behavior may be relatively stable as the firm changes its selling strategy for the new product in response to the general market environment (not related to remanufacturing). We discuss modeling assumptions in Section 3.

From the behavioral/empirical perspective, this paper is among the first to report a study of actual consumer behavior with respect to remanufactured products and analyze a firm’s remanufacturing strategy based on both an analytical model and empirical data. In our study we estimate the fraction of consumers that for a given price difference would switch from purchasing new to purchasing remanufactured product; this fraction plays a central role in our analytical model. We do so in three steps: first, we conduct focus groups to determine the factors consumers consider in making a switching decision; second, we perform conjoint analysis to understand the relative importance of these factors; and third, controlling for the key nonprice factors, we isolate the effect of price and obtain the needed estimate through a survey.

We find that consumers have substantial uncertainty in terminology: are returned / refurbished / remanufactured / rebuilt / reprocessed et cetera products actually the same, or is there a difference? This uncertainty is magnified by not knowing the product’s history: Why was it returned, When, Where, What was wrong? and so forth. As our study shows, consumers would benefit if an independent third party would collect and provide such information (as Carfax does in the auto industry); consumers would then be more willing to purchase refurbished products.

In the study, we find that in the absence of such information, consumers use price to judge product quality. As a result, the fraction of consumers who switch from the new to the remanufactured product has an inverted U-shape – at a very low price, consumers suspect low product quality, and therefore few switch. Note that this behavior cannot be observed if one considers only WTP.

Combining the model and behavioral study, our main findings are the following. First, we show that considering switching behavior and incorporating convex cost that decreases in the number of acquired cores can and often does lead to an optimal solution where the firm procures many more cores than it ends up remanufacturing. We note that this
qualitative conclusion (i.e., that the firm procures cores it does not remanufacture) has been discussed in the literature from cost minimization perspective (e.g., Galbreth and Blackburn 2006), and from the perspective of deferring competitive entry (e.g., Ferguson and Toktay 2005): the firm was acquiring cores in order to deter the entry of a third party (re)manufacturer. Our results demonstrate that firms have economic reasons beyond cost minimization and competitive threats for following such a strategy, thus enhancing its appeal to managers.

Second, we show how various attributes of products and their respective markets (price versus cost of collection and remanufacturing, percentage of falsely returned units, etc.) affect the remanufacturing strategy of the firm, thus providing firms with managerially relevant guidelines regarding their remanufacturing strategies.

Third, we estimate consumer switching behavior and show that it has an inverted U-shape. Most importantly, this implies that by charging a lower price for the remanufactured product, a firm may actually decrease consumer switching and hence minimize high-end demand cannibalization (sales of the new product), while at the same time attracting more low-end (low-price) consumers. It is beneficial for the firm that consumers are uncertain about the quality of remanufactured products and infer quality from price. Otherwise, the firm would have to charge a higher price for the remanufactured product, and with the same degree of new product demand cannibalization, sell fewer remanufactured products to low-end consumers.

Fourth, we compare the solutions for the case when the firm correctly estimates consumer behavior to be inverted-U-shaped (firm-U) and the case when the firm (mistakenly) estimates that consumer behavior is driven by WTP (firm-W). We find that firm-U receives on average 9-13% higher profit, remanufactures under broader conditions, charges a much lower price for remanufactured products and acquires more cores and remanufactures more units. We also find that, quite surprisingly, firms -U and -W react to the size of the low-end segment in the opposite manner: whereas it is optimal for firm-U to increase its price as low-price demand grows, firm-W decreases its price. We stress that these comparisons and intuition are unique to our work, because the described effects cannot be captured if one models consumer behavior using WTP only.

The paper is organized as follows. In Section 2 we position our work in the body of
existing literature. In Section 3 we introduce the model and discuss its main assumptions. Section 4 presents the analysis of the remanufacturing strategy (the relationship between the number of cores to acquire and the number of units to remanufacture). Section 5 presents the study of consumer behavior and shows that the behavior has inverted-U shape. Section 6 studies joint pricing and remanufacturing strategy and discusses the implications of the inverted-U behavior. Section 7 summarizes the paper and discusses opportunities for future work.

2 Literature Review

This work draws on three streams of literature: (i) collection of remanufacturable cores and cost minimization, (ii) market segmentation, competition and cannibalization in remanufacturing, and (iii) behavioral research in operations management in general and remanufacturing in particular.

Collection issues were the focus of early works on remanufacturing, which studied the design and operations of supply chains for reverse logistics, see, for example, Fleischmann et al. (1997) for a review of this earlier literature. More recently, however, collection decisions have been effectively ignored by remanufacturing researchers: in most works, the collection operation is presented as a constraint on the availability of remanufacturable cores, e.g., Ferguson and Toktay (2005). At the same time, as argued by Guide et al. (2003) and Galbreth and Blackburn (2006), whose work, like ours, is motivated by the cell-phone industry, the supply of remanufacturable cores from third-party brokers is essentially unlimited. Collection, therefore, is a decision, rather than a constraint (as is in the case of R discussed above). In Guide et al. (2003) the firm determines the price it will pay for a core of a certain tier (in our terminology), and thus the firm decides simultaneously on both the number and the quality of cores to acquire. The firm also chooses the price of the remanufactured product and maximizes profit, but in doing so, it oversimplifies market segmentation and cannibalization of new product sales. Galbreth and Blackburn (2006), like us, consider the number of cores to acquire as a separate decision. Their work, however, is framed as a cost minimization problem, where demand for remanufactured products is given exogenously. In contrast, we consider
endogenous demand and pricing by linking the acquisition, remanufacturing and pricing (market segmentation) decisions.

Market segmentation, competition, and cannibalization have been studied from a number of angles. For example, Majumder and Groenevelt (2001) and Ferrer and Swaminathan (2005) assume that refurbished products are indistinguishable from new ones and focus on the external competition. Debo et al. (2005), Ferguson and Toktay (2005), Jin et al. (2007) and Souza et al. (2007) assume that consumers value refurbished products less and, as a result, refurbished products are internal competitors to the new product. In addition, some papers also consider external competition from a third-party (re)manufacturer who may also collect the firm’s used products, refurbish them, and offer them on the market, thus cannibalizing the firm’s sales of the new product externally. Their main finding is that the firm may choose to remanufacture or preemptively collect used products to prevent competitive entry, even when the firm would not do so if it would be a monopoly. These works do not consider acquisition decisions, and further assume a fixed marginal cost of remanufacturing.‡ Atasu et al. (2008) consider more elaborate demand patterns by including a “green segment.” We do not consider a green segment, and, as the majority of researchers do, we assume that customers are willing to pay less for the refurbished product; however, we carefully consider the acquisition decisions and allow the cost of remanufacturing to depend upon the number of acquired cores, as suggested by Galbreth and Blackburn (2006) and the case study of R.

Ferguson and Koenigsberg (2007), even though they do not talk about remanufacturing per se, address the management of deteriorating inventory. In their two-period model, the firm in the second period may face an inventory of leftover product, for which customers are willing to pay less than what they would pay for the new product, which is similar to the firm’s putting a refurbished product on the market. They find that the price of the new product is independent of the quality of leftover product (which in their model determines the differential of consumer willingness to pay for it). Motivated by the case

‡An exception is Ferguson and Toktay (2005) who, like us, consider a convex cost of remanufacturing. They, however, do not consider acquisition and do not capture the effect described in Galbreth and Blackburn (2005) as well as in our study of R, where the marginal cost decreases in the number of acquired cores.
of R, we assume a fixed price for the new product, which is consistent with their finding. We note that the majority of works consider pricing of the new product endogenously, which contradicts our experience with R.

Ferguson et al. (2006) consider the management of falsely returned items. In particular, they look at supply chain mechanisms that could reduce false return rates. We do not consider such mechanisms focusing attention on a single firm, but we still analyze the case of false returns as an example for the remanufacturing cost (resulting in a fixed-plus-linear functional form).

Zikopoulos and Tagaras (2007) consider remanufacturing strategy (including collection at possibly more than one site) under the uncertainty in the quality of returns. Our false returns (tier-1/tier-2) cost structure also represents heterogeneity/uncertainty in the conditions of cores, but since this uncertainty is not a primary focus of our paper, we substitute the distribution of conditions with the average outcome (i.e., instead of considering that a core can be tier-2 with probability $\beta$ we assume that a fraction $\beta$ of cores are tier-2). Like some authors mentioned earlier, Zikopoulos and Tagaras document that it may be optimal to collect more cores than firms end up remanufacturing, but they find that “... this behavior is coincidental.” In our setting it is not coincidental: we rigorously prove that a firm may deliberately collect more than it ends up remanufacturing, thus strengthening the managerial appeal of this strategy.

Our paper is also related to the literature that discusses the relationship between price and perceived quality. Such literature is abundant both in academic marketing, and in general business press. For example, The Economist writes§ “... retailers know that people will sometimes turn their noses up at a cheap version of a more expensive product, even if the two are essentially the same.” Survey papers Rao and Monroe (1989) and Völknern and Hofmann (2007) cite over a hundred works on this topic. In conclusion of their paper Völknern and Hofmann write “Managers must be aware that price-quality inferences remain important aspects of consumers behavior and consider them when setting prices... Managers should therefore be cautious when using discounts ... consumers likely make negative price-quality inferences and begin to doubt the quality of the product.” Our paper, as far as we know, is the first work to consider price-perceived quality dependence

in a remanufacturing context (and perhaps in operations management in general).

Finally, with respect to behavioral research in operations management, the body of behavioral work is growing, e.g., Bendoly et al. (2006) and Wu and Katok (2006). A survey of the bodies of knowledge for behavioral operations management is provided in Bendoly et al. (2008). The works involving remanufacturing, however, are very scarce. To our knowledge, Guide and Li (2009), who study cannibalization based on online auctions for computer networking equipment, is the only other work that considers a behavioral experiment in a remanufacturing context. In this paper, we report the results of consumer behavior research (focus groups, conjoint analysis and a survey study) with respect to purchasing remanufactured electronics, and comment on how this research integrates with the modeling results.

3 Model

We assume that there are two products: new (product 2) and remanufactured (product 1). These products are sold at prices $p_2$ and $p_1$, respectively. For the ease of exposition we assume the cost of new product to be zero; an extension to the case with a fixed cost is straightforward (alternatively, $p_2$ can be viewed as a profit margin for the new product). Price $p_2$ is fixed, as we discussed in the introduction and in the case study of R; price $p_1$ is to be determined by the firm.

Demand for the new product – high-end demand – is $D_2$, which we assume to be known deterministically. In the absence of remanufactured product, the firm obtains a “do-nothing” revenue $p_2D_2$; however, given that there is a remanufactured product on the market (which is cheaper but of lower quality) a fraction $\alpha(p_1) \in [\alpha; \bar{\alpha}] \subset [0; 1]$ of the high-end new product’s demand switches to purchase the remanufactured product instead, provided that it is available.

By offering a remanufactured product at price $p_1$, the firm also attracts $D_1(p_1)$ new customers (to which we refer as the low-end demand), who would not purchase the new product; these customers would otherwise choose a different product/brand with lower price. We assume that the new product’s price is selected “optimally” in the sense of $p_2D_2 \geq p_1D_1(p_1)$ for all $p_1 \leq p_2$. Managerially, this condition implies that if the firm de-
cides not to remanufacture then simply decreasing the new product’s price is suboptimal.

If the demand for remanufacturing units exceeds their supply, then these units are allocated on proportion (i.e., on a first-come first-served basis) between the customers who switch from new product and the remanufactured-product-only customers. As a result of this allocation, some switching customers (who were not allocated their most desirable product) – the overflow – continue shopping and purchase the new product.

The firm decides how many units of the new product it collects, \( y \geq 0 \), how many units it remanufactures, \( x \in [0, y] \), and the price of the remanufactured units, \( p_1 \in [0, p_2] \).

For a given \((x, y, p_1)\), the revenue of the firm consists of the following three components:

**Initial sales of the new product** \( p_2(1 - \alpha(p_1))D_2 \);

**Sales of the remanufactured product** \( p_1 \min\{x, D_1(p_1) + \alpha(p_1)D_2\} \);

**Sales of the new product to the overflow customers** \( p_2\alpha(p_1)D_2\left(1 - \frac{\min\{x, D_1(p_1) + \alpha(p_1)D_2\}}{D_1(p_1) + \alpha(p_1)D_2}\right) \)

where the expression for overflow sales reflects the proportional first-come-first-served allocation of the remanufactured units. This overflow component of the model is similar to the simplified model in Ovchinnikov and Milner (2009); please refer to their paper for more detailed discussion. For notational convenience, let \( A(p_1) = D_1(p_1) + \alpha(p_1)D_2 \) represent the total demand for remanufactured products. As is commonly assumed, demand \( A(p_1) \) is a nonincreasing function; i.e., \( \partial D_1/\partial p_1 + D_2 \partial \alpha/\partial p_1 \leq 0 \). This condition obviously holds if \( \alpha \) is itself a decreasing function.

We assume that collecting cores involves a fixed per-unit cost, \( c \), and remanufacturing \( x \) units, given that \( y \) cores have been collected involves a total cost \( r(x, y) \). Then the total profit of the firm is

\[
g(x, y, p_1) = p_2(1 - \alpha(p_1))D_2 + p_1 \min\{x, A(p_1)\} + p_2\alpha(p_1)D_2\left(1 - \frac{\min\{x, A(p_1)\}}{A(p_1)}\right) - cy - r(x, y) = p_2D_2 + p_1 \min\{x, A(p_1)\} - p_2\alpha(p_1)D_2 \frac{\min\{x, A(p_1)\}}{A(p_1)} - cy - r(x, y). \tag{1}
\]

We assume that the remanufacturing cost function \( r(x, y) \) is increasing convex in \( x \) and decreasing in \( y \). We note that this assumption is a generalization of the majority of the previous literature, which considered a linear cost of remanufacturing with a constant
marginal cost. An exception to this is the work of Ferguson and Toktay (2005), who consider a power and, specifically, a quadratic cost function.

In particular, we assume a piecewise linear cost function. Such a function represents the case of so-called false returns and reflects the tiered cost structure in the mini-case of R that we discuss in the introduction. Let $\beta$ be a fraction of false returns. In light of the R’s tiered cost structure, we assume that it costs nothing to remanufacture the first $\beta y$ units; the extension to a nonzero tier-1 marginal cost is trivial. Remanufacturing the remaining units involves a marginal cost $r$, leading to the following piecewise linear cost function:

$$r(x, y) = \begin{cases} 
0, & \text{if } x \leq \beta y; \\
r \times (x - \beta y), & \text{otherwise.} 
\end{cases}$$

(2)

In the remainder of the paper, we discuss the solution(s) to the above model, estimate its key component – switching function $\alpha$ – and discuss managerial implications.

4 Analysis of the Firm’s Remanufacturing Strategy

By the remanufacturing strategy we mean the relationship between the optimal number of units acquired, $y^*$, and the number remanufactured, $x^*$. In this section we show that this relationship holds under some very general conditions, in particular, for any switching function, $\alpha(p_1)$. In the next section we estimate the switching function and determine the values of the remanufacturing strategy parameters, $x^*$ and $y^*$, as well as the price of the remanufactured product, $p^*$. These values clearly depend on the $\alpha$ function, but the relationship between them does not, as we discuss next.

Theorem 1 At optimality the firm follows one of the three strategies: remanufacture nothing (with $x^* = y^* = 0$), remanufacture everything (with $x^* = y^* > 0$), or resell falsely returned units (with $x^* = \beta y^* > 0$). Further, if the firm remanufactures, i.e., $x^* > 0$, then $x^* = \beta y^*$ if $c \leq \beta r$, and otherwise $x^* = y^*$.

Proof: Observe that $g$ is nonincreasing in $x$ for $x \geq A(p_1)$. Thus $x^* \leq A(p_1^*)$.

Consider an arbitrary pair $(p_1, y)$. Observe that if $\beta y > A(p_1)$, then these $(p_1, y)$ cannot be optimal, since by collecting $y' = A(p_1)/\beta$ units, the firm would save $(y - A(p_1)/\beta)c > 0$. 
Therefore $\beta y^* \leq A(p_1^*)$. In this case, observe from (1) that

$$x^*(p_1, y) = \begin{cases} 
0, & \text{if } p_1 - \frac{D_2p_2}{A(p_1)} < 0; \\
\beta y, & \text{if } 0 \leq p_1 - \frac{D_2p_2}{A(p_1)} \leq r; \\
A(p_1), & \text{if } p_1 - \frac{D_2p_2}{A(p_1)} > r.
\end{cases}$$

Suppose that the optimal price, $p_1$, is such that $x^* = \beta y^* < A(p_1)$. Then from (1),

$$g(\beta y, y, p_1) = p_2D_2 + \beta y \left( p_1 - \frac{D_2p_2}{A(p_1)} \right) - cy,$$

which is a linear nondecreasing function of $p_1$ (notice the optimality condition for $x^* = \beta y^*$). Therefore, such $p_1$ cannot be optimal, and since $A(p_1)$ is decreasing, it is thus optimal to increase $p_1$ so that $\beta y^* = A(p_1^*) = x^*$.

Finally, suppose $x^* = A(p_1^*) \geq \beta y$. Then $g = p_2D_2p_1A(p_1^*) - cy - r \times (A(p_1^*) - \beta y)$, and so $\partial g/\partial y = -c + \beta r$. Thus if $c \leq \beta r$ then $x^* = \beta y^*$ and otherwise $x^* = y^*$.

The above theorem has important implications, both analytical and managerial. From the analytical perspective, it implies that provided the firm remanufactures, the decisions about acquisition of cores and remanufacturing/pricing are in fact separable. The firm first decides on how many remanufacturing units to put on the market and how to price them, and then it selects acquisition strategy to minimize remanufacturing cost. From the managerial perspective, and again, provided that the firm offers remanufactured units on the market, the theorem implies that if acquiring cores is not expensive, the firm acquires enough cores so that there are sufficiently many false returned items to satisfy market demand, otherwise it acquires only the number of cores it needs and remanufactures all units that are not false-returns. Returning to the case of R, in the former case R acquires many more cores than it needs, and then selects tier-1 units and resells them (technically speaking it does not do any remanufacturing, just collecting and sorting); in the latter, it actually remanufactures all tier-2 cores.

The results of the Theorem are in agreement with the earlier work of Galbreth and Blackburn (2006) who also showed that the firm may be better off by collecting many more units than it plans on remanufacturing even in the absence of a competitive threat. There is, however, a major difference: Galbreth and Blackburn considered a cost minimization problem and as such started with the case when the firm remanufactures. Before this decision (to remanufacture or not) is made, however, the acquisition and remanufacturing/pricing decisions are clearly not separable, and further, cannot be made without explicitly considering the firm’s revenue. Our paper considers both revenue and cost man-
agement and therefore shows that the strategy of acquiring more cores than “needed” is valid, not just as a cost minimization strategy (which is implied by Galbreth and Blackburn), but also as an overall profit maximization strategy. It therefore has an even stronger managerial appeal.

From the above, the decision of whether to remanufacture or not, and ultimately the firm’s joint pricing and remanufacturing strategy, depends on both revenue and cost components. The revenue, in turn, critically depends on the consumers’ switching behavior. We therefore turn to analyzing this behavior.

5 Study of Consumer Behavior

Consumer behavior with respect to switching from the new to the remanufactured product plays a central role in the firm’s pricing and remanufacturing strategy. Despite its importance, however, little is known about such behavior. In this section we present the results of the study of consumer behavior with respect to the choice between the new and remanufactured products. We discuss how the results of this study could be used to estimate the parameters of the model described above, and comment on the implications for firm’s remanufacturing strategy that follow from both the model and the behavioral study.

Our goal in the study is to estimate the sensitivity of consumer choice to the price difference between the new and remanufactured products (recall that the consumer behavior in our model is described by the function $\alpha(p_1)$, but the price of the new product is fixed at $p_2$, thus, equivalently, $\alpha$ can be viewed as a function of the discount offered for the remanufactured product). To do so, we need to isolate price from other attributes that might influence consumer choice. Therefore, we build the study in three steps. First we conduct focus-group discussions to determine the attributes that are relevant to decisions about purchasing remanufactured consumer electronics products. Second, we perform conjoint analysis to identify the relative importance of various attributes, and in particular, the importance of price in different combinations of other attributes. Finally, based on conjoint analysis results, we control for the main nonprice attributes and survey potential consumers in order to determine price sensitivity.
Subjects in this study are more than 600 graduate students in a U.S. business school, approximately 30% are females and 30% international students. Objects in the study were various consumer electronics products that are familiar to subjects, some of them very similar to those discussed in the introduction in the case study of R. The study was conducted as an independent research elective by a group of four second-year MBA students under the supervision of the author of this paper and a member of the marketing faculty. These students had significant marketing research experience prior to coming to the business school and were concentrating in marketing research during their studies.

We conducted three focus groups discussions with 8 to 10 participants each. Discussions were moderated following a common plan: factors influencing the purchasing of a new product were discussed first, and then a remanufactured option was introduced following the discussion of the factors that the group members deemed important in choosing between the new and remanufactured products.

The findings from the focus-group stage are the following. There are many terms that are commonly applied to remanufacturing: refurbished / remodeled / reconditioned / remanufactured or simply used. Respondents were not sure if these terms meant the same thing or there was a difference. This uncertainty adds to the overall negative perception of remanufactured products, which are believed to be of inferior quality. Participants believe remanufactured products are old or used products that have been rebuilt. There is even skepticism about the “previously opened, but never used” products. Respondents believe that there could be something wrong with the products, but the manufacturer/seller is hiding it. This skepticism leads to an interesting finding, that the degree of uncertainty about remanufactured products could be significantly reduced if there was an independent third-party body that would provide information about the product’s history: when it was bought, why it was returned, what was wrong, etc. Respondents pointed to the existence of such services, for example, in the automotive industry (Carfax), but not in consumer electronics. There was a common belief that if such information were available, that would reduce the uncertainty about remanufactured products and make them a more viable alternative to new products. Product warranty was another important element, as were brand and channel. Respondents were more open to consider a remanufactured product backed by a large warranty, in particular if this warranty came directly from
a manufacturer they know and trust. With respect to brand, interestingly, respondents tended to correlate the quality of new products with uncertain quality of remanufactured products (e.g., if brand D has problems even with the new products, then D’s remanufactured products are perceived to be of even lower quality). These findings are true for all three focus groups.

Based on the above focus-group findings, for the conjoint analysis, we decided to concentrate on a specific product/brand that is very familiar to subjects within consumer electronics: Dell laptop computers. Most subjects purchased Dell laptops as part of the computer bundle with specific software used during their studies; subjects were technically able to purchase refurbished laptops (although no one did that). Further, we selected three levels of product condition and provided very detailed descriptions for them (these descriptions were designed to mitigate the uncertainty described above):

**OOTB – “out-of-the-box”** – systems never used, canceled orders or returns that were never fully booted. This system may have been powered on by a customer before and has been tested and repackaged at the manufacturer’s outlet center.

**CR – “certified refurbished”** – fully refurbished systems which are retested and repackaged to meet original factory specifications. This system may have been powered on by a customer before, used by a consumer before being returned, shipped out to another customer before or may have observable cosmetic blemishes.

**S&D – “scratch-and-dent”** – cosmetic blemishes that do not impact performance. This system may have been powered on by a customer before, used by a consumer before being returned, shipped out to another customer before or may have observable and/or considerable cosmetic blemishes.

We varied warranty for 1, 2, and 3 years, and we varied the discount at 0%, 10%, ..., 90%. Note that although our analytical model is worded in terms of refurbished price $p_1$, since the price of the new product, $p_2$, is fixed, there is a one-to-one correspondence between the discount, $(p_2 - p_1)/p_2$, and price $p_1$. In the behavioral study we used discounts rather than absolute prices because focus groups showed that participants were deciding about purchase based on a percentage discount.
We administered an online choice-based conjoint study using the SSI Web system by Sawtooth Software Inc. (http://www.sawtoothsoftware.com/). Each respondent faced 20 choice tasks, and in each task he or she had to choose among two remanufactured alternatives and a new alternative. We used Sawtooth’s complete enumeration method to generate random choice task designs; this method ensures minimal overlap, good level balance and near perfect orthogonality. The conjoint survey was supplemented by several general questions controlling for age, gender, international status, previous experience with remanufactured products, and perception of remanufactured products. The survey was taken by 141 full-time students and 50 executive students. For executive students the survey also included questions about what car they drive, how they finance their studies, the size of their company and how many levels below CEO they are.

We fit the data to a multinomial logit model using Sawtooth’s add-ons. We first fit the data to a single model; doing that is analogous to estimating total demand for remanufactured products, given by function $A(p_1)$. We immediately saw that the effects of discount, product condition and warranty were highly significant; t-stats of 2.9 and above (in the absolute value). Among the non-price factors, product condition had a substantially greater impact than warranty: attribute importance (the ratio of the range of attribute utilities over the sum of ranges over all attributes) for product condition was approximately two times higher than for warranty (18.7 vs. 11.08). Second, using hierarchical Bayesian estimation we obtained individual partworth utilities and regressed them on the general questions data we collected (e.g., prior experience with refurbished products, which from the focus groups seemed to be an important factor). Interestingly, none of the general questions were statistically significant predictors of individual partworths; t-stats ranged from -0.7 to 0.5. At the same time, examining the individual partworths we noticed that while in general they varied quite substantially, there were certain clear patterns. To capture those patterns we performed latent class estimation; that is, we allowed each individual respondent to belong to one of the distinct segments (classes) and then best fit the partworth utilities for each segment, as well as the probability that an individual belongs to that segment. We experimented with 2, 3, 4, and 5 segments, and found that the Akaike information criterion was lowest for 2 segments (which is in some sense natural – as we argued earlier there exist two distinct segments, the high- and low-
Table 1: Statistics for the two customer segments obtained through latent class estimation of conjoint data. Asterisk at a certain discount level means that the difference between the segments’ utilities is significant at the 95% confidence level.

end where the former is more sensitive to quality and the latter to price). We therefore performed estimation for two segments.

Table 1 presents some statistics of interest for the two segments. Respondents that belong to segment 1 are clearly more sensitive to product quality – attribute importance for product condition is much higher than for segment 2 (and somewhat higher for warranty). In contrast, respondents that belong to segment 2 are more sensitive to price – attribute importance of price is much higher. These segments therefore match with our definitions of the high- and low-end, respectively. Most interestingly, these segments are fundamentally different with respect to sensitivity to discount. In particular, while for discounts of up to about 60% both segments’ utilities are increasing in discount (i.e., the larger is the discount, the more willing the respondents are to purchase refurbished product), for discounts above 60% segment 1 respondents utilities decline; see Table 1. In many ways this already illustrates the central finding of the study: for a non-insignificant number of respondents (consumers), a decrease in price of refurbished product below a certain level does not necessarily translate into an increase in the likelihood of purchase. These respondents are quality-conscious high-end consumers who perceive low priced remanufactured products as low quality and therefore choose to purchase new product.

In sum, with conjoint analysis we saw that many of the non-price factors that focus groups member identified are not significant in predicting the choice of refurbished vs. new product. Warranty and product condition were significant, with product condition being substantially more important. Thus for the final element of our study we controlled for product condition and fixed warranty at 1 year (the same as the warranty for the new product), and created a specialized survey in order to further explore the tradeoff between remanufactured product price and quality.

We use the methodology of Monroe (1990), which was specifically designed to capture
the price-perceived quality effects and is therefore most appropriate for our purposes. For further discussion of this and alternative methodologies refer to the survey by Ofir and Winer (2002). The key element in Monroe's methodology is that consumers have two price thresholds: upper and lower. The upper threshold (question 4 below) is familiar to most operations management researchers as the willingness to pay (WTP): it is the price level above which the product is not worth the money. A rather typical assumption in operations management models is that any price below the WTP leads to a sale. Monroe argued, however, that at a very low price consumers may suspect product quality and refuse to purchase, therefore implying the lower threshold. Monroe suggested asking five questions to estimate these thresholds:

1. At what price would you consider this product to be so inexpensive that you would have doubts about its quality?
2. ... still feel that this product was inexpensive, but have no doubts about its quality?
3. ... begin to feel this product is expensive, but still worth buying because of its quality?
4. ... feel that this product is so expensive that regardless of its quality it is not worth buying?
5. What price would be most acceptable to pay?

We rephrase these questions in terms of the discount offered rather than price, and for each discount level (0-90% in increments of 10%), obtain the percentage of respondents who select a certain level for each question. We received 127 completed surveys: 49 for OOTB, 36 for CR and 42 for S&D product conditions.

Let $s^i(p_1)$ be the proportion of respondents who selected discount level $\frac{p_2-p_1}{p_2}$ in response for question $i$. Let $q^i(p_1) = \sum_{p=0}^{p_1} s^i(p)$ be the cumulative proportion of respondents who selected discount levels higher than $\frac{p_2-p_1}{p_2}$ in response for question $i$. Then as Monroe suggested:

$$\alpha(p_1) = q^4(p_1) - q^2(p_1)$$

That is, the fraction of the high-end customers who switch from a new to remanufactured product is the percentage of respondents who on one hand do not find the remanufactured
Figure 1: Solid lines: percentage of high-end consumers who switch from the new to remanufactured product for different levels of discount (function $\alpha^U$). Dashed lines: percentage of switching consumers if one considers only WTP and disregards price-perceived quality effect (function $\alpha^W$).

product overly expensive, yet on the other do not have doubts about its quality due to discount.

To facilitate the comparison of the optimal policies of the firm that knows the correct estimate of $\alpha$ function, given by (4), and the firm that incorrectly believes that the fraction of switching customers is driven only by the willingness to pay, we differentiate between the true $\alpha$, to which we refer to as the $\alpha^U$ (i.e., $\alpha(p_1) \equiv \alpha^U(p_1)$) and the WTP-based fraction $\alpha^W(p_1)$. From the above $\alpha^W(p_1) = q^i(p_1)$.

Function $\alpha^W$ is clearly decreasing in $p_1$ (equivalently, increasing in the discount level $(p_2 - p_1)/p_2$). Function $\alpha^U$, however, as Monroe’s methodology suggests, has an inverted U-shape. Figure 1 presents both $\alpha^W$ and $\alpha^U$ functions for the three product conditions (dashed and solid lines, respectively). The striking difference between the two is evident: for example, for the OOTB product condition at 40% discount, 100% of subjects found the product’s price below their WTP. A “traditional” WTP-based approach would suggest that all customers would switch; hence $\alpha^W = 1$; however, 59% of subjects doubt product quality: If this is indeed an OOTB (never fully booted, canceled order) product, then why is the firm selling it at 40% discount? – there must be something wrong with it, but the firm does not tell. As a result only 41% switches; hence $\alpha^U = 0.41$.

With this, the goal of the study of consumer behavior is accomplished – we obtained
an estimate of the function \( \alpha^U(p_1) \), as well as the estimate of the WTP-based function \( \alpha^W \). The main differences between the \( \alpha^U \) and \( \alpha^W \) functions are summarized in the following observations (which are evident from the definitions of \( \alpha^U \) and \( \alpha^W \) and from Figure 1):

Observation 1 The following relationships hold for all \( p \in (0, p_2) \):

(a) \( \alpha^W(p) \geq \alpha^U(p) \geq 0 \);

(b) \( \alpha^W(p) \) is decreasing in \( p \) (equivalently, increasing in the discount level \((p_2 - p_1)/p_2\)), \( \alpha^U(p) \) has an inverted-U shape;

(c) \( \partial \alpha^U/\partial p \geq \partial \alpha^W/\partial p \).

Next we use these observations and the estimates we obtained to solve the model and discuss the implications of knowing the inverted-U switching function.

6 Joint pricing and remanufacturing strategy and implications of the inverted-U consumer behavior

In this section we discuss the value of recognizing that consumer behavior has an inverted-U shape. To do so we compare the optimal joint pricing and remanufacturing decisions for two firms. Firm-U believes that consumer behavior is inverted-U-shaped and is described by function \( \alpha^U \); firm-W believes that behavior is described by \( \alpha^W \) (while it is actually \( \alpha^U \)). Otherwise the firms face identical problems. We use superscripts \( U \) and \( W \) to differentiate between these firms throughout the section.

We start by discussing when firms-U and -W remanufacture.

**Theorem 2** Suppose there exists \( \mathbf{p}_1 \geq \min \left[ \frac{c}{\beta}, c + (1 - \beta)r \right] \) for which \( \alpha^U(\mathbf{p}_1) = 0 \), but \( D_1(\mathbf{p}_1) > 0 \). Then firm-U always remanufactures.

**Proof**: Suppose that firm-U charges the price \( \mathbf{p}_1 \) and decides to offer \( A(\mathbf{p}_1) \) remanufactured units. Since \( \alpha^U(\mathbf{p}_1) = 0 \), \( A(\mathbf{p}_1) = D_1(\mathbf{p}_1) \). From the proof of Theorem 1, if the firm remanufactures, then its acquisition decision, \( y^* \), is separable, and can be expressed as \( y^*(x, p_1) \). Thus \( g_U(x_U, y_U^*, p^*_U) \geq g(D_1(\mathbf{p}_1), y^*(D_1(\mathbf{p}_1), \mathbf{p}_1), \mathbf{p}_1) = p_2 D_2 + D_1(\mathbf{p}_1)(\mathbf{p}_1 - ...) \)
min[c/β, c + (1 − β)r] ≥ p_2D_2, i.e., the firm’s profit is higher that the do-nothing profit of p_2D_2 and hence it is not optimal not to remanufacture.

We note two things. First, the condition \( p_1 > \min[c/β, c + (1 − β)r] \) has an intuitive meaning: it implies that the price of a remanufactured product must be higher than the average cost of (optimally) collecting and remanufacturing it (observe that the expression in the minimum operator is analogous to \( c ≤ βr \) in Theorem 1). If the price is lower than the average cost, the firm obviously will not remanufacture. Second, observe that the existence of the price \( p_1 \) at which \( α^U(p_1) = 0 \) is not that surprising either: in our study, for all three product conditions, discount levels of 60-80% and above satisfy this condition. Combining these two conditions, the Theorem therefore states that (as a sufficient condition) for the firm-U to remanufacture, the cost of collecting/remanufacturing should not exceed these 20-40%, respectively, of the new product’s price.

For firm-W, in order to provide similar result we assume that \( α^W(p_1) \) is a linear function. Note that \( α^W(p_1) ≤ 1 \) and thus in practice it is at best piecewise linear (e.g., see Figure 1). However, \( p_1 \) so low that \( α^W(p_1) = 1 \) clearly cannot be optimal because \( p_2D_2 ≥ p_1D_1 \). Thus linearity is a reasonable assumption for \( p_1 \) such that \( α^W(p_1) < 1 \). Further, since by definition \( α(p_2) = 0 \), we can therefore assume \( α^W(p_1) = b(p_2 − p_1) \) for some coefficient \( b \). For simplicity, we will also assume that \( D_1 \) is linear, and further, because \( D_1(p_2) = 0 \) we assume that \( D_1(p_1) = d(p_2 − p_1) \) for some coefficient \( d \). We assume linearity for the rest of the paper. This allows us to solve firm-W pricing and remanufacturing problem in closed form. Table 2 and Theorem 3 present the results (the proof of the Theorem is in the appendix).

**Theorem 3** The optimal price, number of cores acquired and number of remanufactured units put on the market for firm-W are presented in Table 2.

That is, for firm-W there are four different solutions that correspond to five different intervals for the parameters of the model (a “do-nothing” solution appears twice). What do these solutions mean, and why is a particular solution optimal for the corresponding intervals of parameters?

Observe that condition 1 for solutions 1 and 2 implies that the the firm is dealing with a product that is cheap to collect (\( c \) is small), but expensive to remanufacture (\( r \)
Table 2: Solutions to the firm-W problem (assuming linear demand and switching).

<table>
<thead>
<tr>
<th>Condition 1</th>
<th>Condition 2</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{c}{\beta} + p_2 \frac{d+2bD_2}{2(d+bD_2)} \leq r)</td>
<td>(c \leq \frac{dp_2}{d+bD_2})</td>
<td>(x^* = \frac{dp_2-c/\beta(d+bD_2)}{2}, y^* = x^<em>/\beta,) (p^</em>_1 = \frac{p_2+c/\beta}{2} + \frac{bD_2p_2}{2(d+bD_2)})</td>
</tr>
<tr>
<td>(2)</td>
<td>(c \geq \frac{dp_2}{d+bD_2})</td>
<td>(x^* = 0, y^* = 0,) (p^*) is arbitrary</td>
</tr>
<tr>
<td>(\frac{c}{\beta} + p_2 \frac{d+2bD_2}{2(d+bD_2)} \geq r)</td>
<td>(c \geq r) and (c + (1-\beta)r \leq \frac{dp_2}{d+bD_2})</td>
<td>(x^* = \frac{dp_2-(c+(1-\beta)r)(d+bD_2)}{2}, y^* = x^<em>,) (p^</em>_1 = \frac{p_2+c+(1-\beta)r}{2} + \frac{bD_2p_2}{2(d+bD_2)})</td>
</tr>
<tr>
<td>(4)</td>
<td>(c \geq r) and (c + (1-\beta)r \geq \frac{dp_2}{d+bD_2})</td>
<td>(x^* = 0, y^* = 0,) (p^*) is arbitrary</td>
</tr>
<tr>
<td>(5)</td>
<td>(\frac{c}{\beta} \leq r)</td>
<td>(x^* = \frac{dp_2-r(d+bD_2)}{2}, y^* = x^<em>/\beta,) (p^</em>_1 = \frac{p_2+r}{2} + \frac{bD_2p_2}{2(d+bD_2)})</td>
</tr>
</tbody>
</table>

is large). Condition 2 for solution 1 corresponds to the case when the percentage of false returns is high (\(\beta\) is large), whereas for solution 2, it corresponds to the reverse case, when the percentage of false returns is low. With this intuition, solutions 1 and 2 seem quite natural: if the firm is dealing with a product that is cheap to collect but expensive to remanufacture, then if the percentage of false returns is high, the firm should collect many units (since it is cheap to do so) and not remanufacture any units (since it is expensive), but rather resell those that were falsely returned. Since there are many such units, collecting and reselling is beneficial for the firm. If, however, there are few such units (low percentage of false returns in solution 2), then reselling them does not cover the cost of collecting substantially more units, and so the firm does nothing.

On the contrary, condition 1 for solutions 3-5 says that the firm’s product is expensive to collect but cheap to remanufacture. Furthermore, condition 2 for solution 3 says that the product is also expensive (\(p_2\) is high). In this case, the firm can sell remanufactured units at a higher price, and so it chooses to remanufacture all units it collects (solution 3). If, on the other hand, the product is not expensive, then the firm chooses to do nothing (solution 4). Finally, solution 5 presents the case with balanced costs, where the firm engages in the same reselling strategy as in solution 1, but it resells fewer units at a higher price since the units are more expensive to collect (observe how solution 1
compares to solution 5 in light of condition 2 for solution 5).

In sum, Theorems 2 and 3 discuss the optimal solutions to firm-U and firm-W problems. Next, we compare these solutions, and emphasize the value of knowing that consumer switching behavior is inverted-U. We start by showing that at optimality, firm-U necessarily receives higher profit than firm-W.

**Theorem 4** \( g_U^* \geq g_W^* \).

**Proof:** Observe that at \( x = A(p_1) \), \( g \) decreases in \( \alpha \) (for a fixed \( p_1 \) at which firm still remanufactures): \( \frac{\partial g}{\partial \alpha} = -\partial_2D_2 + (p_1 - \min[c/\beta, c + (1 - \beta)r])D_2 \leq 0 \) (note that if firm remanufactures then \( p_1 \) is larger than the expression in the min operator, but \( p_1 \leq p_2 \)).

Consider the optimal decisions of firm-W, \( (x_W^*, y_W^*, p_{1W}^*) \), and suppose that firm-U decides to charge price \( p_{1W}^* \). If firm-W does not remanufacture w.l.o.g. we can assume \( p_{1W}^* = p_2 \) and the result of the Theorem is trivial. If firm-W remanufactures, then \( p_{1W}^* < p_2 \). Further, in that case from the proof of Theorem 1 acquisition decision, \( y_U^* \), is separable, and can be expressed as \( y_U^*(x_U, p_{1U}) \), where the optimal price and quantity of firm-U satisfy \( A(p_{1U}^*) = x_U^* \). That is, if firm-U decides to charge price \( p_{1W}^* \) then it remanufactures \( A_U(p_{1W}^*) = D_1(p_{1W}^*) + \alpha_U^*(p_{1W}^*)D_2 \) units.

Because \( \alpha_U^*(p) \leq \alpha_W^*(p) \) for all \( p \) and because \( g \) is decreasing in \( \alpha \) for a \( p \) that satisfies \( x = A(p) \), \( g_W^* \leq g_U(A(p_{1W}^*), y_U^*(A(p_{1W}^*), p_{1W}^*), p_{1W}^*) \leq g_U^* \).

From the above proof firm-U receives higher profits even if it charges the same price as firm-W, \( p_{1W}^* \). This happens because firm-U recognizes that at price \( p_{1W}^* \) it will cannibalize less of the new product’s demand, and hence its overall demand for remanufactured units will be smaller. Firm-U therefore puts fewer remanufactured units on the market, and both sells more new products (at the higher price), and saves on remanufacturing and acquisition. At the same time, firm-U can clearly do even better if it charges a different price, \( p_{1U}^* \), as we discuss next.

**Theorem 5** \( p_{1U}^* \geq p_{1W}^* \).

**Proof:** It is easy to observe that at a given price \( p_1 \),
\[
\frac{\partial g_W}{\partial p_1} - \frac{\partial g_U}{\partial p_1} = \left( \frac{\partial \alpha^W}{\partial p_1} - \frac{\partial \alpha^U}{\partial p_1} \right) (-p_2D_2 + (p_1 - \min[c/\beta, c + (1 - \beta)r])D_2) \geq 0
\]
because the first expression is negative by Observation 1 and the second is also negative because \( p_2 \geq p_1 \).

Further, because by assumption \( \alpha^W \) and \( D_1 \) are linear (and by definition decreasing),
\[
\frac{\partial^2 g_W}{\partial p_1^2} = 2 \left( \frac{\partial D_1}{\partial p_1} + \frac{\partial \alpha^W}{\partial p_1} D_2 \right) \leq 0,
\]
i.e., \( g_W \) is concave in \( p_1 \). Thus, from the former, at the optimal price \( p_{1U}^* \), \( \partial g_W / \partial p_1 \geq 0 \), and from the latter \( \partial g_W / \partial p_1 \) is decreasing. Hence \( \partial g_W / \partial p_1 = 0 \) at the price greater than \( p_{1U}^* \), i.e., \( p_{1U}^* \leq p_{1W}^* \).

Contrasting the results of Theorems 4 and 5, from the former, firm-U can get higher profit than firm-W because if firm-U charges the same price as firm-W it will remanufacture less and hence save on cannibalization and remanufacturing cost. But from the former, firm-U will in fact not charge the same, price; rather, it will charge a smaller price. By doing so firm-U can decrease cannibalization even further (e.g., observe that \( \alpha^U \) goes to zero for large discount levels on Figure 1), and more effectively discriminate between the high-end new product’s customers, to whom it will sell at new products at price \( p_2 \), and the low-end remanufactured-product-only customers, to whom it will sell remanufactured products at price \( p_1 \).

It is interesting to note that nothing can be said (in general) about the relationship between \( x_{1U}^* \) and \( x_{1W}^* \) (correspondingly \( y_{1U}^* \) and \( y_{1W}^* \)). This is because even though \( p_{1U}^* \leq p_{1W}^* \) and \( \alpha^U(p) \leq \alpha^W(p) \), we also know that \( \alpha^U \) is first increasing and then decreasing. Thus it is not clear whether \( \alpha^U(p_{1U}^*) \leq \alpha^W(p_{1W}^*) \) or not; if this relationship holds then \( x_{1U}^* \geq x_{1W}^* \), and otherwise \( x_{1U}^* \leq x_{1W}^* \). We note that in our numerical simulations, when both firms remanufactured, the former was always the case.

### 6.1 Numerical Simulations

To further facilitate comparison between firm-U and firm-W we perform a series of numerical simulations. We fix \( p_2 = 100 \) and \( D_2 = 100 \); this can be done w.l.o.g. because of scaling. We set \( D_1(p_1) = d(p_2 - p_1) \) and vary \( d = \{0.05; 0.1; 0.2, 0.3, 0.5, 1, 1.5, 2, 3\} \). With respect to the cost of remanufacturing, given by (2), we consider \( c = 1, 3, r = c \times \{2; 5; 8\} \) and \( \beta = \{0.1; 0.2; 0.3; 0.4; 0.5\} \). This leads to 270 combinations of cost and demand parameters.
For each combination, we solve the models for firms-U and -W. For firm-W $\alpha^W(p_1) = \min[1, c^W(p_2 - p_1)]$, where $c^W = 3.76, 2.82, 2.99$ for the OOTB, S&D and CR conditions, respectively. These coefficients were obtained by best-fitting the piecewise linear function to the data for the WTP-based switching behaviors, dashed lines in Figure 1.

For firm-U we tested two approaches. In the first approach, we use the empirical $\alpha^U$ function for the values of $p_1 = 0, 10, 20, ..., 100$ as per the data on Figure 1, and for $p_1$ between the tested values we use linear interpolation. In the second approach, we fit the data to a Beta-density function multiplied by a coefficient to account for the fact that $\alpha^U$ must not integrate to one. The parameters of the fit are as follows: for OOTB: Beta$(3.2, 8.6)/3.75$, for S&D: Beta$(3.05, 7.05)/4.45$, and for CR: Beta$(5.2, 11.05)/4.55$. The sum of squares errors for the three conditions are $0.0090$, $0.023$, and $0.0033$, respectively, and the maximum absolute errors are $0.05$, $0.085$, and $0.05$. As such, the quality of fit is good, and as a result, the difference between the solutions we obtained using these approaches is small. We report the interpolated case below.

For both models we solve for $p^*, x^*, y^*$. We use complete enumeration search with $p_1 \in \{0, 1, 2, ..., 100\}$ and $y \in \{5, 10, ..., 500\}$, for each $p_1, y$ we solve for the optimal $x^*$. Computation is implemented in Mathematica on a standard desktop PC.

First we observe when firms-U and -W remanufacture. In our simulations, firm-U always remanufactures. This is not surprising in light of Theorem 2 and because as per Figure 1 at low $p_1$ (high discount) $\alpha^U$ is zero. Cost of remanufacturing in our simulations is at maximum $3 + 0.9 \times 24 = 25.6$ and at $p_1 = 25.6$ we have $\alpha^U$ of either zero, or almost zero. Thus the conditions of Theorem 2 are basically met and firm-U therefore remanufactures.

That is not the case, however, for firm-W. In our simulations, firm-W does not remanufacture in 86 of the 270 cases. These cases correspond to those with small $d$ parameter, i.e., with small and inelastic low-price segment. From Theorem 3 and Table 2 these are the cases 2 and 4 (see condition 2) for which it is optimal for firm-W to do nothing. Thus, again, the result by itself is not that surprising. Overall, however, the different way firms-U and -W react to the size of the low-price segment is interesting, and we discuss it separately, after discussing the differences in prices, quantities and profits.

Second, we observe the difference in prices and quantities. To have a “fair” basis for comparison, we only consider the 184 cases when firm-W remanufactures. Over those
cases, the price of firm-U is 48 (for CR condition) to 53 (for OOTB condition) dollars lower than that of firm-W. Since the new product’s price is 100 such a difference suggest a major difference in the policies of the firms. Firm-W is not charging a lower price because it is afraid of cannibalizing new product’s demand. Visually, it is operating on the left side of the plot on Figure 1. To the contrary, firm-U knows that it can combat cannibalization by a drastic price reduction. Visually, it operates on the right side of the \( \alpha^U \) curve, where price is small. By doing so it more effectively segments the market and sells new product at a high price to the high-end new product consumers, and sells remanufactured product at a low price to low-end consumers. We also observed that the quantities of remanufactured products, and as a result, the quantities of acquired cores, are higher for the firm-U: 19 (for CR) to 26 (for OOTB) units higher on average.

Third, as Theorem 4 suggests, firm-U must receive a higher profit. In our simulations, firm-W by optimizing its remanufacturing activities increased its profit on average by 5.5% (for OOTB) to 7% (for SD) over a do-nothing profit. On top of that, firm-U was able to further increase profits additionally by 9% (for SD) to 13.5% (for CR) on average. That is, knowing that consumer behavior has an inverted-U shape and optimizing pricing and remanufacturing accordingly triples the benefits of remanufacturing. Further, the total profit increase from remanufacturing activities for firm-U was between 16% and 20% on average over all 270 scenarios we considered, which is also a very significant figure. In sum, knowing that consumer behavior has an inverted-U shape, and adjusting the pricing and remanufacturing strategy accordingly, clearly pays off.

We conclude this section by discussing the differences in firms responses to the size of the low-price customer segment, Figure 2. The main observation is that as the size of low-price demand segment increases, firm-W decreases its price, while firm-U increases its price. Further, when demand is very small, firm-W does not remanufacture (observe that the curve for firm-W starts at \( d = 0.2 \)), while firm-U charges a very low-price. What causes such a drastic difference?

In short, it the inverted-U shape of switching behavior. One of the key drivers of the firm’s remanufacturing policy is cannibalization of new product sales by the remanufactured product. If the consumer switching behavior has an inverted U-shape, then the same cannibalization can be obtained by either giving a small discount or a large one.
Figure 2: Optimal prices of firm-U and firm-W as a function of the low-price demand parameter, $d$ (for the OOTB case with $c = 1$, $r = 2$, $\beta = 0.1$).

- for example, from Figure 1, 40% of consumers will switch at discounts of either 15% or 40%. But by giving a large discount – that is, by charging a low $p_1$, firm-U attracts more low-price bargain-hunting consumers who would not buy its new product at all. Therefore, with the U-shaped consumer behavior, the firm can choose the lower price, and while obtaining the same cannibalization, bring more low-end consumers.

Thus, when there are few low-end customers (i.e., $d$ is small), in order to capture most of them, firm-U charges a very low price (simultaneously decreasing high-end demand cannibalization). Firm-W cannot charge a low price since it is afraid to cannibalize all of its new product’s demand and hence does nothing. Visually, “starting” point for firm-U is at the right of the inverted-U curve on Figure 1, and starting point for firm-W is at the left. As $d$ increases, firms move in the opposite directions. Firm-W decreases the price, brings in some low-end customers and cannibalizes some new product’s demand, but it sells remanufactured products at a high price. Firm-U is increasing the price, losing the proportion of the low-price demand it serves, but because the number of low-price customers increases, it may be actually gaining the low-end demand. It also increases cannibalization, but at a smaller rate than firm-W. Finally, it also increases the price, so the increase in cannibalization is offset by an increase in per-unit revenue. This explains the behaviors we observe on Figure 2, and, once again, points out to a very significant
difference in the decisions the firm makes if it knows about the inverted-U consumer behavior.

7 Conclusions

This paper considers firms’ joint pricing and remanufacturing strategy from two perspectives – modeling and behavioral/empirical. In the model, we consider the revenue component, where the firm selects the price and quantity of remanufactured products in the market to balance the cannibalization of new product sales with the additional sales of remanufactured products. We also consider the cost component, where the firm balances the acquisition of remanufacturable cores with the number of remanufactured products it wishes to put on the market.

The central role in the revenue component of the model is played by the fraction of consumers who, for a given price difference, switch from the new to the remanufactured product. We argue that using such a fraction provides more flexibility than the standard approach of using willingness to pay (WTP). Further, we present the behavioral study where we show that, in fact, the realistic consumer behavior cannot be explained by the WTP alone – rather, when the remanufactured product’s price is very low consumers infer product quality from price and few decide to switch (the WTP approach would suggest that there should be many switching customers when the price is low). In the cost component of the model, we assume a convex cost that decreases in the number of acquired cores – this modeling setup reflects heterogeneity in acquired products’ conditions and was motivated by the case study that we present in the introduction.

We solve the model and discuss how various attributes of the products and their respective markets (price versus cost of collection and remanufacturing, percentage of falsely returned units, etc.) affect the remanufacturing strategy of the firm. This provides firms with intuitive and managerially relevant guidelines with respect their remanufacturing strategy. We show that in all cases the firm’s strategy is either to “remanufacture nothing,” “remanufacture everything that was collected,” or “resell falsely returned products.” The latter strategy, under which the firm procures many more cores than it ends up remanufacturing, was known in the literature, but it was motivated by either cost minimization...
(in the models that did not consider revenue component), or by the fear of competitive entry – the firm was acquiring the cores in order to deter the entry of a third-party (re)manufacturer. As our paper shows, however, firms may have economic reasons beyond cost minimization and competitive threats for following such a strategy, which from a managerial perspective, strengthens its practical appeal.

Complementing the model, we present the behavioral study and estimate the switching consumer behavior. We show that the switching fraction has an inverted U-shape. This implies that by charging a lower price for the remanufactured product, the firm can decrease consumer switching, and hence minimize demand cannibalization of the new product; at the same time it can attract more low-end consumers. Our results also shed a new light on the importance of the low-price segment for the pricing and remanufacturing strategy. Indeed, the above can be done only if by charging a lower price the firm can attract sufficiently many new low-price-only customers. Otherwise, the firm must charge a higher price for the remanufactured product, and with the same degree of new product demand cannibalization, sell fewer units. This implies that the optimal strategy of the firm can be drastically different depending on the size/elasticity of the low-price segment, which in turn implies that firms should dedicate more effort in researching that segment.

Based on both the model and the empirical study we compare the solutions for the firm that correctly estimates its inverted-U-shaped consumer behavior (firm-U) with the solutions for the firm that (incorrectly) believes that its consumers behave according to the WTP (firm-W). We find that knowing that consumer behavior is inverted-U and optimizing pricing and remanufacturing accordingly, effectively triples the benefits of remanufacturing. Firm-U receives on average 9-13% higher profit than firm-W (which in turn receives 5-7% higher profit than do-nothing), remanufactures under broader conditions, charges a much lower price for remanufactured products and acquires more cores and remanufactures more units. We also find that, quite surprisingly, firms -U and -W's react to the size of the low-price segment in the opposite manner: whereas it is optimal for firm-U to increase its price as low-price demand grows, firm-W decreases its price. We stress that these comparisons are intuition are unique to our work, because the described effects cannot be captured if one models consumer behavior using only WTP.

In terms of the future work, the paper offers several possibilities. First extension could
consider competition. For example, if firms-U and -W as defined in this paper were competitors, would knowing inverted-U consumer behavior provide even more value? Another extension could consider demand uncertainty and substitution. Extensions dealing with supply chain implications might also be considered.

It could also be interesting to consider introduction of remanufactured products as a mechanism to create/change product lines. For example, returning to the case study of R, a $139 remanufactured version of a $399 PDA could be cannibalizing not only the sales of this PDA, but also the sales of the “regular” telephones priced around $139. At the same time, since much of R’s revenue comes from sales of voice and data plans, it could be that consumers who purchase PDAs are more likely to spend more, for example, on data plans. Thus cannibalizing regular telephone sales could be in fact profitable for R overall. Studying such dynamics could be also of interest for future research.

Acknowledgement

The author gratefully acknowledges the help of Professors Marian Moore and Ron Wilcox, as well as Class of 2008 MBA students Jennifer Chick, Katherine Horsman, Michael Rabinovitz and Damon Petite, who helped to supervise and implement, respectively, the consumer behavior study.

References


Appendix

First, consider a simplified case with linear cost, \( x = y \) and \( r(x, y) = rx \), i.e., all collected units are remanufactured and remanufacturing additional unit incurs a constant marginal cost (rate) \( r \geq 0 \) per unit. For notational convenience we omit the \( y \) variable in this initial discussion.

As we discuss throughout the paper,

\[
x^*(p_1) = \begin{cases} 
0, & \text{if } p_1 - \frac{p_2 \alpha(p_1) D_2}{A(p_1)} - r \leq 0; \\
A(p_1), & \text{otherwise.}
\end{cases} \tag{5}
\]

Substituting (5) into (1) and rearranging with \( x = y \) in mind, we obtain

\[
g^*(p_1) = p_2 D_2 + ((p_1 - r)A(p_1) - p_2 \alpha(p_1) D_2)\quad. \tag{6}
\]

Let \( \Delta(p_1) = (p_1 - r)A(p_1) - p_2 \alpha(p_1) D_2 \). From (5) and (6) the firm engages in remanufacturing if \( \Delta(p_1) \geq 0 \). This condition can be rewritten as

\[
(p_1 - r)D(p_1) \geq (p_2 - (p_1 - r))\alpha(p_1)D_2,
\]

and therefore has a simple managerial explanation. It states that the firm will remanufacture if the revenue potential from new customers who would purchase only remanufactured product offsets the cannibalization of the sales of the new product caused by introducing the remanufactured product, corrected for the cost of remanufacturing, \( r \).

From (6) maximizing \( g^*(p_1) \) is identical to maximizing \( \Delta(p_1) \). Observe that \( \Delta'' = 2D' + (p_1 - r)D'' + 2D_2 \alpha' - (p_2 - (p_1 - r))D_2 \alpha'' \leq 0 \) since \( D' \leq 0, \alpha' \leq 0 \) and \( D'' = \alpha'' = 0 \) by linearity assumption. Therefore \( \Delta(p_1) \) is concave and so the optimal price of remanufactured product, \( p_1^* \), satisfies the first-order condition

\[
A(p_1^*) + (p_1^* - r)D_1'(p_1^*) - (p_2 - (p_1 - r))D_2 \alpha'(p_1^*) = 0. \tag{8}
\]

Finally, if \( \Delta(p_1^*) > 0 \) then the firm remanufactures (and collects) \( y^* = x^*(p_1^*) = A(p_1^*) = D_1(p_1^*) + \alpha(p_1^*) D_2 \) units and offers them for sale at price \( p_1^* \). Otherwise the firm does not remanufacture, \( x^* = y^* = 0 \) and \( p_1^* \) is irrelevant.

Utilizing the assumption that \( D_1(p_1) = d(p_2 - p_1) \) and \( \alpha(p_1) = b(p_2 - p_1) \) we can obtain closed-form solutions. Solving FOC (8) we obtain

\[
p_1^* = \frac{p_2 + r}{2} + \frac{bD_2 p_2}{2(d + bD_2)}. \tag{9}
\]
and therefore
\[ x^*(p_1^*) = A(p_1^*) = \frac{dp_2 - r(d + bD_2)}{2}. \] (10)

Since \( A(\cdot) \geq 0 \), it follows that if
\[ r \geq \frac{dp_2}{d + bD_2}, \] (11)
then \( x^* = 0 \) and the firm does not remanufacture and obtains a “do-nothing” profit \( D_2p_2 \).
Otherwise the firm remanufactures \( \frac{dp_2 - r(d + bD_2)}{2} \) units and sells them at price \( p_2 + \frac{bD_2p_2 - dp_2}{2(d + bD_2)} \).
By doing so, the firm obtains a profit \( g^* = p_2D_2 + \left( \frac{dp_2 - r(d + bD_2)}{4(d + bD_2)} \right)^2 \).

Now consider general cost. Observe that both the optimal price and the optimal quantity depend on the number of units collected, \( y \). Specifically if no units are collected, i.e., if \( y = 0 \), then obviously \( x = 0 \) and the price is irrelevant. Otherwise, the firm collects some units, \( y \), and therefore it could choose to remanufacture. Therefore it has to select the optimal price, \( p_1^*(y) \) for the case when \( x^*(p_1^*(y), y) = A(p_1^*(y)) \) for a given \( y \) (we study the problem of selecting the optimal \( y \) later).

By linearity of demand and switching, \( A(p_1^*(y)) = (d + bD_2)(p_2 - p_1^*(y)) \). Substituting it into (1) and differentiating in \( p_1 \) we obtain:
\[ \frac{\partial g^*(A(p_1), y, p_1)}{\partial p_1} = (d + bD_2)(p_2 - p_1) + bD_2p_2 + (d + bD_2) \frac{\partial r(x, y)}{\partial x} \big|_{x = A(p_1)} \] (12)
and
\[ \frac{\partial^2 g^*(A(p_1), y, p_1)}{\partial p_1^2} = -2(d + bD_2) - (d + bD_2)^2 \frac{\partial^2 r(x, y)}{\partial x^2} \big|_{x = A(p_1)} \leq 0 \] (13)
because \( r(x, y) \) is convex in \( x \) for all \( y \).
That is, \( g^* \) is concave in \( p_1 \) for all \( y \) when \( x^*(p_1^*(y), y) = A(p_1^*(y)) \). Therefore, in this case the optimal price \( p_1^*(y) \) satisfies
\[ p_2 - 2p_1^* + \frac{bD_2p_2}{d + bD_2} + \frac{\partial r(x, y)}{\partial x} \big|_{x = A(p_1^*)} = 0. \] (14)

Given the (implicit) functions for both the optimal number of remanufactured units, \( x^*(p_1^*(y)) = A(p_1^*(y)) \), and the optimal price of remanufactured unit, \( p_1^*(y) \), as per (14) both as functions of the number of units collected for remanufacturing, \( y \), we could de-
etermine the optimal \( y \) by solving the following nonlinear program:

\[
(y - NLP) \quad \text{max } g(x^*(p^*_1(y)), y, p^*_1(y))
\]

\[
s.t. \quad A(p^*_1(y)) \leq y
\]

\[
x^*(p^*_1(y)), y, p^*_1(y) \geq 0
\]

where \( p^*_1(y) \) is given by (14) and profit function \( g(\cdot) \) is given by (1). Finally if \( g^* > p_2 D_2 \), i.e., the optimal profit from remanufacturing is greater than the do-nothing profit, then the optimal solution holds, otherwise \( y^* = x^* = 0 \), and \( p_1 \) is irrelevant.

As per Theorem 1, if \( x^* > 0 \) then \( x^* = y^* \) or \( x^* = \beta y^* \). The former case was analyzed initially, so suppose \( x^* = \beta y^* \). By the definition of \( A(\cdot) \)

\[
p^*_1(y) = p_2 - \frac{\beta y}{d + bD_2}.
\]

Substituting these expressions into the profit function and differentiating over \( y \), we obtain:

\[
\frac{\partial g^*(y)}{\partial y} = -c + \frac{\beta dp_2 - 2y\beta^2}{d + bD_2}
\]

and

\[
\frac{\partial^2 g^*(y)}{\partial y^2} = -\frac{2\beta^2}{d + bD_2} < 0.
\]

That is, the profit function is concave, and so it is optimal to collect \( y^* = \frac{-c(d + bD_2) + \beta dp_2}{2\beta^2} \) units. Therefore from (17)

\[
p^*_1 = \frac{p_2 + c/\beta}{2} + \frac{bD_2p_2}{2(d + bD_2)}
\]

\[
x^*_1 = \frac{dp_2 - c/\beta(d + bD_2)}{2}.
\]

Since \( \beta \leq 1 \), this solution automatically satisfies constraint (15), i.e., that the firm cannot remanufacture more units than it collects. For the nonnegativity constraint (16) we must have \( y^* \geq 0 \) (which implies \( x^* \geq 0 \)) and \( p^* \geq 0 \). From (20) the latter always holds, while the former demands \( c \leq \beta \frac{dp_2}{d + bD_2} \). Finally, this solution should satisfy the range for prices, i.e., since \( 0 \leq p_1 - \frac{bD_2p_2}{d + bD_2} \leq r \) from (3), we must have

\[
\frac{c}{2\beta} + p_2 \frac{d + 2bD_2}{2(d + bD_2)} \leq r.
\]

The condition \( 0 \leq p_1 - \frac{bD_2p_2}{d + bD_2} \leq r \), leading to (22), suggests that such a policy would be feasible when it is relatively cheap to collect the units (\( c \) is small), but it is expensive
to remanufacture them \((r \text{ is large})\), while the percentage of false returns, \(\beta\), is high. Since this is not always the case, we are considering the remaining case where the optimal price \(p_1^*\) is such that \(p_1^* - \frac{bD_2p_2}{d+bD_2} \geq r\).

In this case, observe that for a fixed \(y\), the model resembles that with the linear cost of remanufacturing. Therefore from (9) and (10) \(p_1^* = \frac{p_2 + r}{2} + \frac{bD_2p_2}{2(d+bD_2)}\) and \(x^*(p_1^*) = A(p_1^*) = \frac{dp_2 - r(d+bD_2)}{2}\) respectively, so long as \(x^* \leq y\). Since these solutions are independent of \(y\), and for given \(x\) and \(p_1\) the profit function is linear in \(y\), we obtain \(\frac{\partial g(x^*,y,p_1^*)}{\partial y} = -c + \beta r\).

Hence if \(c/\beta \leq r\), then the firm would like to collect as many units as possible and so \(y^* = A(p_1^*)/\beta\), where \(p_1^*\) is given by (9). Otherwise the firm would like to collect as few units as possible, and hence the solution satisfies \(x^* = y^*\). In this case by the same logic as initially:

\[
p_1^* = \frac{p_2 + c + (1 - \beta)r}{2} + \frac{bD_2p_2}{2(d+bD_2)}
\]

and therefore

\[
x^*(p_1^*) = A(p_1^*) = \frac{dp_2 - (c + (1 - \beta)r)(d+bD_2)}{2}
\]

and this solution holds if \(x^* \geq 0\), which is equivalent to \(c + (1 - \beta)r \leq \frac{dp_2}{d+bD_2}\). Otherwise \(x^* = y^* = 0\) and \(p^*\) is arbitrary.