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DESIGN OF PRICE AND ADVERTISING ELASTICITY MODELS

Introduction

The marketing mix that a manager may deploy can affect the sales of a product and can be categorized under the traditional four Ps of marketing (product, price, promotion, and placement). But the perennial question managers face concerns the combination of these different marketing mix variables that will give them maximized sales, highest share, lowest inventory, or maximized margins. Quite often, these questions are answered by historical data: for example, past sales or market share for different levels of expenditures on these marketing mix variables. In this note, we consider the design of models that allow managers to obtain robust price and advertising elasticity estimates.

Consider the following scenario: Belvedere vodka was introduced in the United States in 1996. This vodka traced its roots back to the Warsaw suburb of Żyrardów, Poland, and its production process went back more than 600 years. Lately, it had begun to observe a decline in its overall share of the vodka market. The company suspected the cause to be new market entrants that were capturing market share with effective advertising. To sustain the growth rate and defend its share from the competition, Belvedere was considering two options: increasing its advertising expenditure and/or reducing the pricing. Such a scenario is very common for most brands during the various stages of their brand (or product) life cycles. The first step toward solving this issue is to estimate the elasticity of a brand to its price and advertising.

Price Elasticity of Demand

Pricing is one of the most critical variables that marketers have problems with. Based on common sense, consumers tend to buy more of a product as its price goes down, and using the same logic, they will buy less if the price goes up. Price elasticity of demand is a measure to show the responsiveness of the quantity demanded of a good (or service) to a change in its price; it gives the percentage change in quantity demanded in response to a 1% change in price

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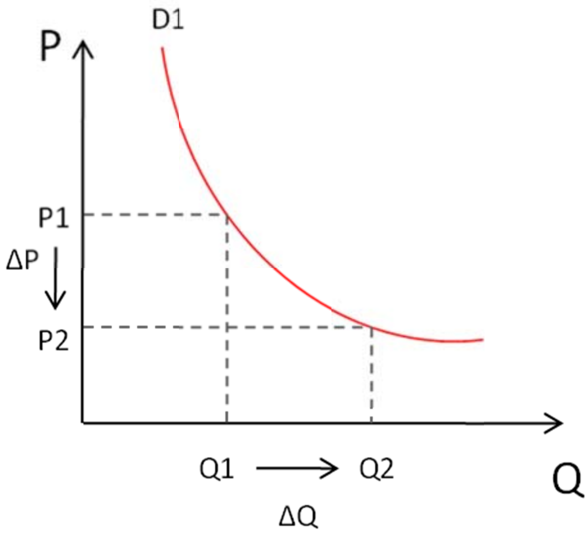
(holding constant all the other variables in the marketing mix).[1] A product with a price elasticity above 1 is said to be elastic, as changes in demand are relatively large compared with changes in price. Correspondingly, a product whose elasticity goes below 1 is deemed inelastic.

Figure 1: Price elasticity of demand.



Price elasticity can be derived as the ratio of change in quantity demanded (%Δ*Q*) and percentage change in price (%Δ*P*).

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Price Elasticity of Demand (PED) can be calculated using **Equation 1**:

$$\text{PED} = [\text{Change in Sales/Change in Price}] \times [\text{Price/Sales}] = (\Delta Q/\Delta P) \times (P/Q) \quad (1)$$

Or if we have a sample of historical sales and price data, then we can regress the sales against price, and the coefficient of this regression will give price elasticity as shown in **Equation 2**:[2]

$$\text{PED} = \text{Coefficient of price when } Ln(\text{sales}) \text{ is regressed on } Ln(\text{price}) \quad (2)$$

Here, we are assuming that the *Ln*-*Ln* model (i.e., dependent *Ln*(sales) regressed on independent *Ln*(price)) gives us a better linear model, which historically has been the case with most models, such as that in **Equation 3**:

$$Ln(\text{sales}) = \alpha_1 + \beta_1 \times Ln(\text{price}) + \varepsilon_1 \quad (3)$$

where β_1 represents the price elasticity in the above case and ε_1 is the random error term drawn from a normal distribution (the standard assumption in a linear regression model).

[1] This is essentially never the case—as will be explored in more detail later in this note.

[2] The coefficient and price elasticity are by definition not the same, but they are very closely related to each other, and in most of the cases, the coefficient is a close proxy for the elasticity.

Assuming consumers are rational and reasonably informed, the coefficient (and hence the price elasticity) should be negative. Therefore, the phrase "greater price sensitivity" means more negative price elasticity, and similarly "less price sensitivity" means less negative price elasticity.

Refer to **Exhibit 1** for Belvedere's sales and price data and the regression results. With a regression coefficient of −1.259, we can say that price elasticity of sales for Belvedere is high (i.e., its customers are fairly price-sensitive). Reducing price may have a positive impact on sales. This model suggests that a price decrease of 1% may result in 1.259% sales increase of 9L cases of Belvedere vodka.

Advertising Elasticity of Demand

The advertising elasticity of demand (AED) is a measure of the responsiveness in the demand of a product to changes in the level of advertising. It can be calculated by using **Equation 4**:

$$\text{AED} = [\text{Change in Sales/Change in Advertising}] \times [\text{Price/Advertising}] = \boldsymbol{(\Delta Q/\Delta A) \times (A/Q)} \quad (4)$$

Let us suppose that the total advertising exposure in period 1 was \$100 and total sales were 200 units. Then, in period 2, the advertising was increased to \$125 and the total sales were 300 units. Here, an advertising spend increase of \$25 resulted in a sales increase of 100 units. So, AED = (100 ÷ \$25) × (200 ÷ \$100) = 4. In other words, a 1% increase in advertising results in a 4% increase in sales. Similar to the procedure for price elasticity, the basic formulation to estimate advertising elasticity is to run a regression of log of sales (or market share) on log of advertising. The coefficient of the log of advertising will be the estimate of advertising elasticity (**Equation 5**):

$$Ln(\text{sales}) = \alpha_2 + \beta_2 \times Ln(\text{advertising}) + \epsilon_2 \quad (5)$$

where β_2 is the advertising elasticity of demand and ε_1 is the random error term drawn from a normal distribution.

All other factors remaining equal, an increase in advertising is expected to result in a positive shift in demand and hence a positive advertising elasticity. AED can be utilized by a firm to make sure its advertising expenses are in line, though an increase in demand may not be the only desired outcome of advertising.

Refer to **Exhibit 2** for regression results of Belvedere's sales and advertising data. A regression coefficient of −0.013 and low t-stat value suggest that changing advertising expenses may have no impact on Belvedere's sales or that the change cannot be predicted. Elasticities (or sensitivities) can be used for short-term advertising effects[3]—values less than 0 imply negative returns to advertising and greater than 1 imply the firm is underadvertising. So the value should range from 0 to 1. In our simple regression model above, we took advertising expenditure as one simple independent variable by combining expenditures for all possible media. But different media (e.g., print, display, in-store, television) may have varied impact on the demand of a product based on its characteristics. More analysis is required to study the impact of different media, and if required, more than one variable should be incorporated in the regression model to get a better-fitting model and help the marketing manager decide on the advertising expenditure—both the total amount and its distribution across different media.

Building a Comprehensive Model

If both PED and AED are significant, the regression model should include both price and advertising as independent variables (**Equation 6**):

$$Ln(\text{sales}) = \alpha + \beta_1 \times Ln(\text{price}) + \beta_2 \times Ln(\text{advertising}) + € \quad (6)$$

"Bias" is a commonly used term to describe the impact of omitted variables. It is used where there are systematic differences in the estimated elasticity (due to errors in estimation, not environmental differences) and the true elasticity in the market. This bias may be caused by omission of variables, which may be correlated with those included in the equation. The decision to include or omit certain variables in the model other than price and advertising will therefore depend on the correlation of a variable with the dependent variable and its correlation with other independent variables. If advertising elasticity is higher than the true value, then it is said to be a positive bias, but if it is lower than the true value, then it is called a negative bias. Conversely, if price elasticity is more negative than the true value, then it is said to be a positive bias, and if it is less negative, then it is called a negative bias (see **Figure 1**).

[3] In the case of multiplicative models, the coefficients were elasticities, whereas in the case of linear models, elasticities can be estimated by multiplying the regression coefficient by the ratio of means of the dependent variable and the advertising measure.

Figure 1. Building a model.

If the true model is as follows:

$Ln(Y) = \alpha_0 + \alpha_1 \times Ln(\text{price}) + \alpha_2 \times Z + \epsilon$,

but we estimate the model to be

$Ln(Y) = \beta_0 + \beta_1 \times Ln(\text{price}) + \epsilon$,

the true value of coefficient β_1 will be the sum of the estimated coefficient β_1 and the bias

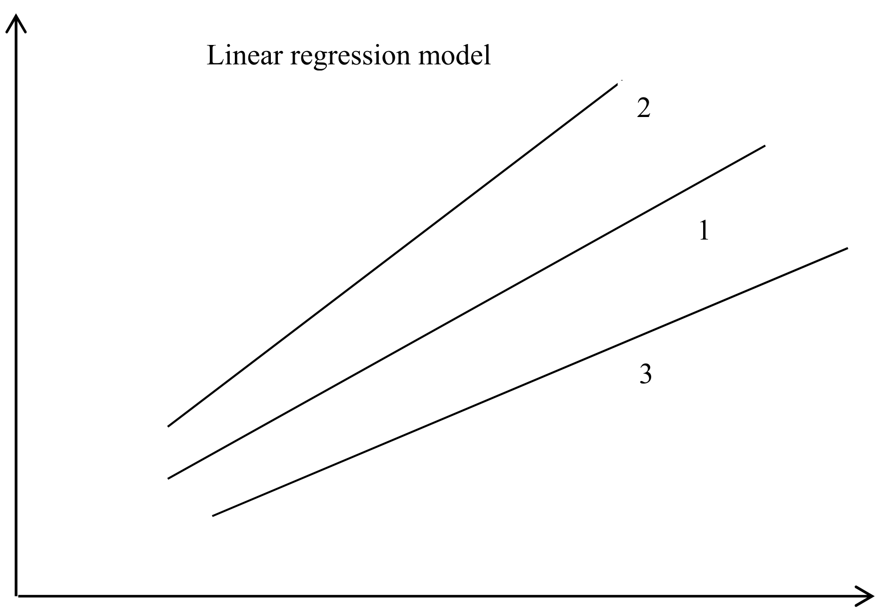
$\beta_1^{\text{true}} = \beta_1 + \text{bias}$.

If r is the covariance between independent variables, *Ln*(price) and Z,

then the bias can be proven to be the product of the coefficient of the omitted variable (α_2) and some function of covariance of independent variables [$f(r)$]:

$\text{Bias} = \alpha_2 \times f(r)$.

- If the dependent variable is not related to the omitted variable, then there is no bias (=0).
- If the included independent variable (i.e., *Ln*(price)) is not correlated to the omitted variable (i.e., covariance is zero), then there is no bias (=0).
- **If α_2 (correlation between the omitted variable and the dependent variable) and r (covariance between independent variables) are of same sign, then the bias is positive.**
- **If α_2 and r are of different signs, then the bias is negative.**



1 = true model, with all variables included

2 = estimated model with omitted variables—positive bias

3 = estimated model with omitted variables—negative bias

Source: Created by case writer.

Below are some major factors that would need to be included in a comprehensive marketing mix model for price and advertising elasticity.

Product quality

If consumers are even minimally informed about the product quality, then the better-quality product would be able to command higher prices. With this assumption, the correlation coefficient for a regression model on price and quality will be positive. Therefore, if higher-quality products also sell more, the omission of quality from the model would lead to positive bias of price elasticity. This means that the estimated price elasticity in a model without product quality would be more negative than a model that includes product quality.

Distribution

The more widely the product is available to customers, the better the sales of that product. But the relationship between distribution and price (and between distribution and sales) is not straightforward. Firms with high-priced brands typically have selective (or exclusive) distribution channels. If this strategy holds, the omission of distribution would lead to less negative price elasticity (i.e., a negative bias).

Brand life cycle

As a brand matures, consumers' knowledge about that brand (e.g., deals, prices, comparable, availability) increases. Also, the early adopters of a brand are less price-sensitive. Therefore, price elasticity tends to increase (i.e., become more negative) over the life cycle of a brand.

Time series data versus cross-sectional data

Price elasticity for a brand will have two components: (1) a within-brand component, a measure of sensitivity to prices of a particular brand over time and (2) a between-brands component, a measure of sensitivity to differences between brands. Because consumers mostly respond to prices at the point of purchase, using only a snapshot of data across brands without any time variation leaves out the within-brand component of elasticity. If the within-brand component is weak (less negative), then this sort of data aggregation over time would lead to a positive bias in price and advertising elasticity. Price elasticity would be more negative and advertising elasticity would be more positive if the model used data across brands for a single time period. When prices and advertising are included over time, it is better if the frequency of the time series reflects the product's purchase cycle. For example, for consumer packaged goods, the price and advertising elasticities are more accurate if the sales, pricing, and advertising decisions are sampled every week so that they are reflective of consumer's typical grocery trip frequency.

Carryover effect of advertising

Advertising rarely has an “immediate” impact on sales. If we take into account the effect of advertising on sales for the current period, more often than not, those effects would be in the form of spikes and they would be relatively small (i.e., quite fragile) as compared with other marketing variables. Some research indicates that the current effect of price is 20 times larger than the current effect of advertising. The portion of advertising that retains its effect and affects consumers even beyond the period of its exposure is known as the carryover effect. Depending on the product type, consumer segment, and firm’s strategy, there could be several reasons for this carryover effect: delayed consumer response due to their backup inventory, delayed exposure to the ad, shortage of retail inventory, and so on. Therefore, to account for the total effect of advertising, include both the current effect and all the carryover effect.

The Koyck model provides a way to capture the carryover effect of advertising: It enhances the basic linear marketing mix model, by including a lagged dependent variable as an additional independent variable. So, as per the enhanced model, sales of the current period depend on sales of the prior period and all the independent variables that caused prior sales, plus the current values of the same independent variables.

If the original model (before Koyck) was as shown in **Equation 7**:

$$Ln(Y_t) = \alpha + \beta_1 \times Ln(A_t) + \beta_1 \times B_t + \epsilon_t \qquad (7)$$

then **Equation 8** is the enhanced model (by Koyck):

$$Ln(Y_t) = \alpha + \boldsymbol{\lambda} \times \boldsymbol{Ln(Y_{t-1})} + \beta_1 \times Ln(A_t) + \beta_1 \times B_t + \epsilon_{t.} \qquad (8)$$

In this model, β_1 captures the current effect of advertising, while $\beta_1 \times \lambda/(1 - \lambda)$ can be calculated to be the carryover effect of advertising. The higher the value of factor λ, the longer the effect of advertising will be. Similarly, the smaller the value of λ, the shorter the effects of advertising will be (i.e., sales depend more on current advertising). The total effect of advertising is the sum of current and carryover effects; that is, $\beta_1/(1 - \lambda)$.

If the advertising effects are positively correlated from one period to the next (i.e., the last period’s advertising has a positive correlation with current period’s advertising), and if the past advertising has a positive correlation with the current period’s sales, then the omission of the carryover effect will result in a positive bias.

Contextual factors

Another factor that may come in to play is the disposable income of consumers in a region where a product is being sold. Consumers in countries (or regions) with high disposable income may be less price-sensitive. If so, then higher income would lead to lower price elasticity (i.e., less negative). At the same time, better-informed customers in a region (as well as stronger regulations and antitrust laws) may lead to increased price sensitivity.

Overall, the exogenous variables (e.g., GNP and sociodemographics such as average family income, family size) generally have a positive correlation with sales, and their exclusion could have a positive bias on the model. The regional context may also have a correlation with advertising, for example, due to differences in preferences, production cost structures, and restrictions.

Table 1 summarizes the impact of bias due to the omission of different variables from the marketing mix:

Table 1. Impact of bias in price and advertising elasticity.

Factor	**Bias in Price Elasticity**	**Bias in Advertising Elasticity**
Product Quality	+	
Distribution	–	
Brand Life Cycle—Early	+	
Absolute Sales	+	
Time Series	–	–
Include Carryover		+
Contextual Factors (income, family size, etc.)		+

Other factors to consider while designing the model include:

Promotion

Promotional activities can take one of two forms: (1) increasing product awareness through displays, campaigns, demonstrations, and so forth or (2) incentivizing consumers to try a company's products through coupons, rebates, and so on. Normally, firms tend to run the incentive programs along with their strategy of charging higher prices: higher prices for existing customers and rebates to acquire new customers. In such a case, prices and promotions would be positively correlated. On the other hand, the other form of promotions (increasing awareness) is generally used concurrently with lower prices. The goal is to maximize consumer awareness, and in this case, the prices and promotions would be negatively correlated. In either case, inclusion of promotion characteristics is necessary to obtain a better distinction between price effects and promotion effects on sales.

Competition

The price elasticity tends to be more sensitive if the firm compares the price of its products with that of its competitors'. Consumers tend to consider relative price rather than absolute price when opting for a specific brand. Therefore, an increase in price may not negatively impact sales if the competition also raises prices in the same period. Following the same logic, if a firm fails to respond to a price change from the competition, the choice may affect its sales (negative for price decline by competition and positive for price increase).

Share versus volume

If sales volume is used as a dependent variable and advertising as independent, sales may be gained from a competitor (existing market) and sales may be gained from new customers (market expansion due to advertising). But if instead of sales, market share is used as a dependent variable, market expansion is eliminated as a possible reason.[4] As a result, the models using share (instead of sales) should normally have smaller elasticity.

Conclusion

A marketing mix model can be a strategic asset for a firm. Developing a good model requires knowledge of advanced statistics as well as a deep understanding of consumer behavior and the business context. Response models provide a good tool to aid marketing mix decisions. They give managers a way to assess the relative importance of their different marketing mix options. The product line may be the most effective marketing mix option, followed by distribution, price, and promotion.[5] Managers should, however, be aware that response models assume that the market a company will face in the future would remain unchanged as compared with the past (or the time frame used to estimate the advertising and price elasticities). Price and advertising elasticities are an accurate reflection of consumer preferences, competitor reactions, the number of brands, firm strategy, and other market factors during the time of data collection. Expectations of returns from a firm's marketing mix decisions must be informed by anticipated competitor actions and changes in the consumer preferences and competitive landscape. For this reason, it would also be a good idea to periodically update the marketing mix models and re-estimate price and advertising elasticities.

[4] With market share as a dependent variable, the impact of advertising will appear in both numerator and denominator.

[5] Berk M. Ataman, Harald J. Van Heerde, and Carl F. Mela, "The Long-Term Effect of Marketing Strategy on Brand Sales," *Journal of Marketing Research* 47, no. 5 (2010): 866–82.

Exhibit 1

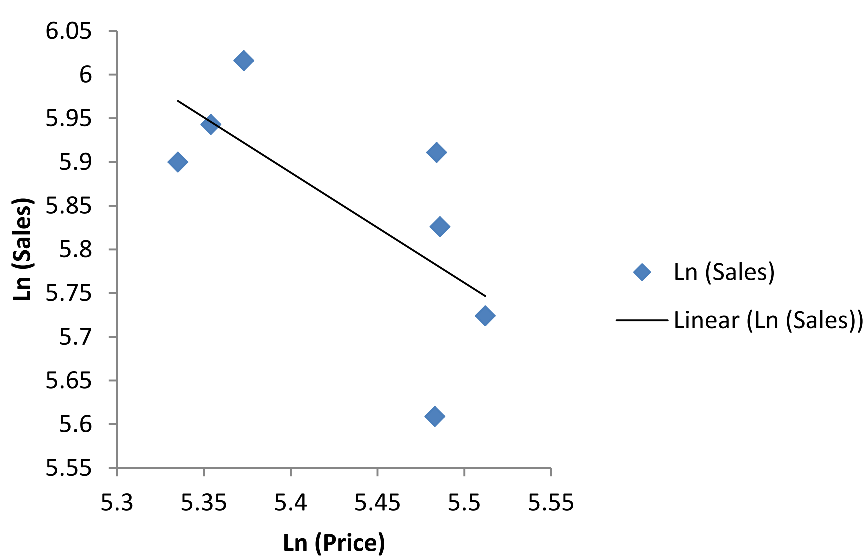
DESIGN OF PRICE AND ADVERTISING ELASTICITY MODELS

Regression of *Ln*(Sales) versus *Ln*(Price)

Year	Sales (thousands of units)	*Ln*(Sales) (thousands of units)	Price (dollars)	*Ln*(Price) (dollars)	Advertising (thousands of dollars)	*Ln*(Advertising) (thousands of dollars)
2007	410	6.016	215.44	5.373	20486.1	9.93
2006	381	5.943	211.45	5.354	2923.5	7.98
2005	365	5.900	207.45	5.335	4826.3	8.48
2004	369	5.911	240.87	5.484	13726.6	9.53
2003	339	5.826	241.33	5.486	10330.2	9.24
2002	306	5.724	247.55	5.512	13473.6	9.51
2001	273	5.609	240.48	5.483	9264.6	9.13

Regression Statistics	
Multiple R	0.67536
R Square	0.45611
Adjusted R Square	0.34733
Standard Error	0.11269
Observations	7

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	12.686	3.340	3.798	0.013
Ln (Price)	−1.259	0.615	−2.048	0.096



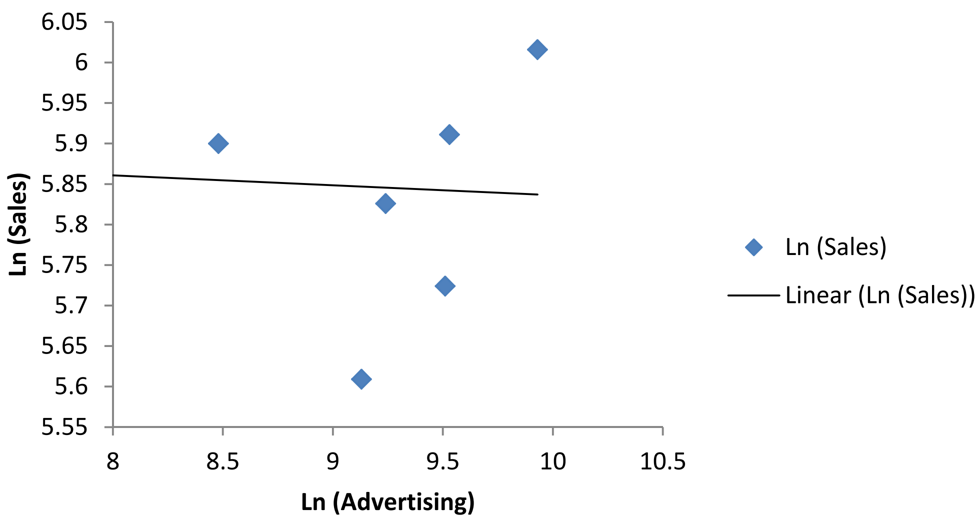
Source: Created by case writer.

Exhibit 2

DESIGN OF PRICE AND ADVERTISING ELASTICITY MODELS

Regression of *Ln*(Sales) versus *Ln*(Advertising)

Regression Statistics				
Multiple R	0.06102			
R Square	0.00372			
Adjusted R Square	−0.19553			
Standard Error	0.15252			
Observations	7			
	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	5.963	0.850	7.018	0.001
Ln (advertising)	−0.013	0.093	−0.137	0.897



Source: Created by case writer.

References

Assmus, Gert, John U. Farley and Donald R. Lehmann. “How Advertising Affects Sales: Meta-Analysis of Econometric Results,” *Journal of Marketing Research* 21, no.1, February 1984, 65–74.

Parsons, Leonard J. “A Rachet Model of Advertising Carryover Effects,” *Journal of Marketing Research* 13, no. 1, February 1976, 76–79.

Tellis, Gerard J. “The Price Elasticity of Selective Demand: A Meta-Analysis of Econometric Models of Sales,” *Journal of Marketing Research* 25, no. 4, November 1988, 331–41.

Tellis, Gerard J. “Modeling Marketing Mix,” chapter 24 in *The Handbook of Marketing Research: Uses, Misuses, and Future Advances*. Thousand Oaks, CA: Sage Publications, 2006, 506–22.